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SMOOTHING OF COMPLEX SURFACE TRIANGULATIONS WITH EFFICIENT LOCAL NODE MOVEMENTS

R. Montenegro*, **J.M. Escobar, G. Montero and E. Rodríguez**

University Institute for Intelligent Systems and Numerical Applications in Engineering
University of Las Palmas de Gran Canaria, Campus Universitario de Tafira
35017 Las Palmas de Gran Canaria, Spain
E-mail: rafa@dma.ulpgc.es - Web page: http://serv05.iusiani.ulpgc.es/proyecto0507/html/

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Summary. *This work presents a new method for quality improvement of a given surface triangulation maintaining its topology and controlling the surface approximation. The main advantage is the transformation of the 3-D problem into 2-D local parametric ones without constraints. So, the local objective function, which is defined by using algebraic quality measures, contains a barrier which automatically prevents the method from constructing unacceptable meshes. Besides, the proposed method has only one free parameter which controls the gap between the optimized mesh and the reference surface.*

1 INTRODUCTION TO THE SMOOTHING METHOD

A new problem arose us in mesh generation of 3-D domains defined over complex terrains [1, 2] including chimneys. In order to prevent a loss of details of the original surface, we did not allow movement of nodes placed over the terrain during the optimization procedure of the 3-D triangulation. This aspect motivates the introduction of a new technique to improve the quality of triangular meshes defined on surfaces.

For 2-D or 3-D meshes the quality improvement [3] can be obtained by an iterative process in which each node of the mesh is moved to a new position that minimizes an objective function [4]. This function is derived from a quality measure of the local mesh and presents a barrier in the boundary of the *feasible region* (set of points where the free node could be placed to get a *valid* local mesh, that is, without *inverted elements*). This barrier has an important role because it avoids the optimization algorithm to create a tangled mesh when it starts with a valid one. Nevertheless, objective functions constructed by algebraic quality measures are only directly applicable to inner nodes of 2-D or 3-D meshes, but not to its boundary nodes [5, 6]. To overcome this problem, a local mesh, $M(p)$, sited on a surface Σ, is orthogonally projected on an optimal local plane P in such a way that it performs a valid local mesh $N(q)$. Therefore, it can be said that $M(p)$ is *geometrically conforming* with respect to P [7]. Here p is the free node on Σ and q is its projection on P. The optimization of $M(p)$ is got by the appropriated optimization of $N(q)$. To do this we try to get *ideal* triangles in $N(q)$ that become equilateral in $M(p)$. In general, when the local mesh $M(p)$ is on a surface, each triangle is placed on a different plane and it is not possible to define a feasible region on Σ. Nevertheless, this region is perfectly defined in $N(q)$.

To construct the objective function in $N(q)$, it is first necessary to define the objective function in $M(p)$ and, afterward, to establish the connection between them. A crucial aspect for this construction is to keep the barrier of the 2-D objective function. This is done with a suitable approximation in the process that transforms the original problem on Σ into an entirely two-dimensional one on P.

The optimization of $N(q)$ becomes a two-dimensional iterative process. The optimal solutions of each two-dimensional problem form a sequence $\{\mathbf{x}^k\}$ of points belonging to P. We have checked in many numerical test that $\{\mathbf{x}^k\}$ is always a convergent sequence. It is important to underline that this iterative process only takes into account the position of the free node in a discrete set of points, the points on Σ corresponding to $\{\mathbf{x}^k\}$ and, therefore, it is not necessary that the surface is smooth. Indeed, the surface determined by the piecewise linear interpolation of the initial mesh is used as a reference to define the geometry of the domain.

If the node movement only responds to an improvement of the quality of the mesh, it can happen that the optimized mesh loses details of the original surface. To avoid this problem, every time the free node p is moved on Σ, the optimization process only allows a small distance between the centroid of the triangles of $M(p)$ and the underlaying surface (the true surface, if it is known, or the piece-wise linear interpolation, if it is not).

There are several alternatives to the previous method. For example, Garimella et al. [8] develop a method to optimize meshes in which the nodes of the optimized mesh are kept close to the original positions by imposing the Jacobians of the current and original meshes to be also close. Frey et al. [9] get a control of the gap between the mesh and the surface by modifying the element-size (subdividing the longest edges and collapsing the shortest ones) in terms of an approximation of the smallest principal curvature radius associated to the nodes. Rassineux et al. [10] also use the smallest principal curvature radius to estimate the element-size compatible with a prescribed gap error. They construct a geometrical model by using the Hermite diffuse interpolation in which local operations like edge swapping, node removing, edge splitting, etc. are made to adapt the mesh size and shape. More accurate approaches, that have into account the directional behavior of the surface, have been considered in by Vigo [11] and, recently, by Frey in [12].

2 EXAMPLE

In this section, the proposed technique is applied to smooth the mesh of a scanned object. In particular, we have applied the optimization technique to a mesh obtained from http://www.cyberware.com/. The object is a screwdriver (see Figure 1) with 27150 triangles and 13577 nodes. Note the poor quality of the original mesh in several parts. The average quality (measured with the quality metric based on the condition number [4]) is increased from 0.822 to 0.920 in four iterations, see Figure 2. The worst 500 triangles increases its average quality from 0.486 to 0.704. It is important to remark that the original geometry is almost preserved in the optimization process.

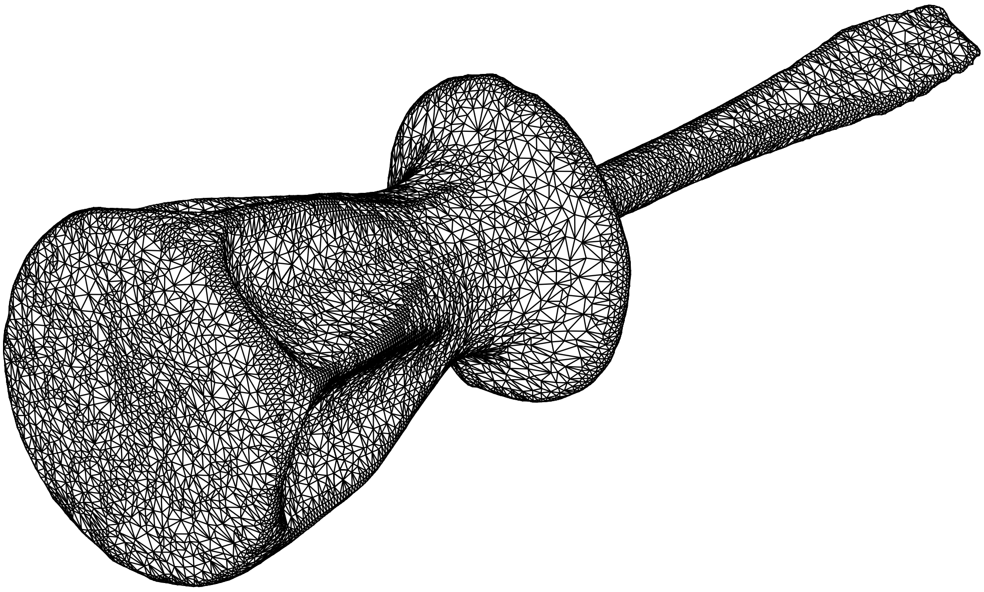

Figure 1: Original mesh of a screwdriver obtained from http : //www.cyberware.com/

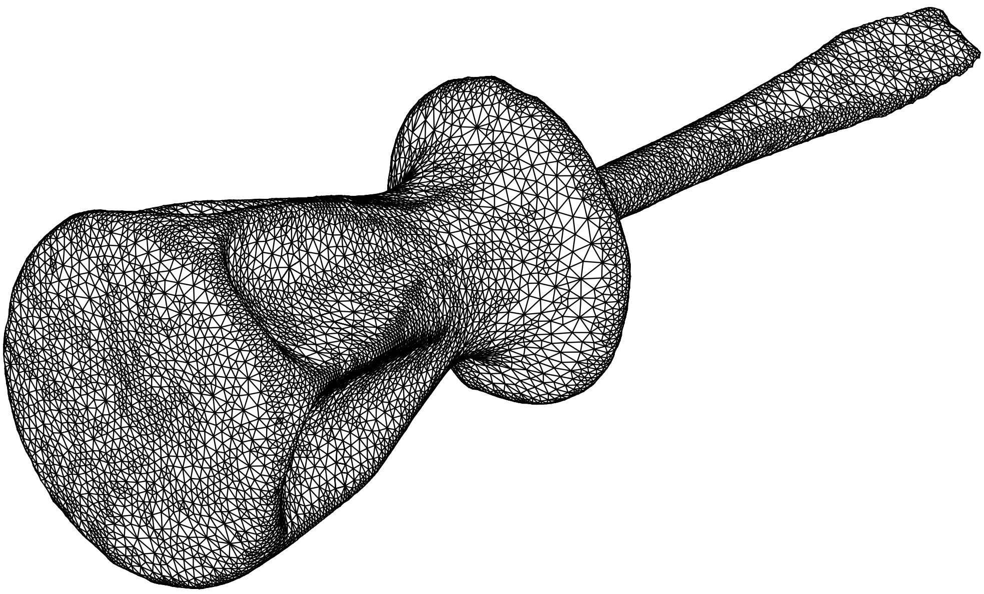

Figure 2: Optimized mesh of the screwdriver after four iterations of our smoothing procedure

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