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Some advances in open problems of isogeometric analysis

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Isogeometric analysis (IGA) has arisen as an attempt to unify the fields of CAD and classical finite element methods. The main idea of IGA consists in using for analysis the same functions (splines) that are used in CAD representation of the geometry. The main advantage with respect to the traditional finite element method is a higher smoothness of the numerical solution and more accurate representation of the geometry. IGA seems to be a promising tool with wide range of applications in engineering. However, this relatively new technique have some open problems that require a solution. In this work we present our results and contributions to this issue.

First of all, the main drawback of using B-splines and NURBS for geometric design and IGA is the impossibility to perform local refinement due to its tensor product structure: knot insertion propagates through the domain. T-splines were introduced by Sederberg et al. as an alternative to NURBS. Based on the idea of admitting meshes with T-junctions (T-meshes) and inferring local knot vectors by traversing mesh edges, T-splines have provided a promising tool for geometric modelling that allows to perform local refinement without introducing a large number of superfluous control points. However, in order to be used for numerical analysis, the functions must meet some requirements: linear independence, polynomial reproduction property, local supports and possibility to perform local adaptive refinement. This issue has been the object of numerous research works in recent years.

Here we describe our strategy [1] for constructing tensor product spline spaces over hierarchical T-meshes with quad- and octree subdivision scheme. The proposed technique includes some simple rules for inferring local knot vectors to define spline blending functions. These rules allow to obtain, for a given T-mesh, a set of cubic spline functions that span a space with nice properties: the functions are linearly independent and C^2-continuous, they can reproduce cubic polynomials, and spaces spanned by nested T-meshes are also nested. In order to span spaces with these properties applying the proposed rules, the T-mesh should fulfill the only requirement of being a *0-balanced* mesh. The straightforward implementation of the proposed strategy can make it an attractive tool for its use in geometric design and isogeometric analysis. We illustrate some examples of its application. Optimal rates of convergence are obtained during adaptive refinement performed for a Poisson problem over complex 2D and 3D geometries.

Another drawback of IGA is that it requires a global parameterization of the computational domain. CAD provides only surface representation of the geometry. So it is necessary to have a robust and effective method to obtain appropriate volume parameterization of computational domain from its surface representation. We have developed a method [2], based on the ideas of [3], to define a global parameterization of the domain. The algorithm obtains, as a result, a high quality parametric transformation between 2D objects and the parametric domain, the unit square. The key of the method lies in defining an isomorphic transformation between the parametric and physical T-mesh finding the optimal position of the interior nodes by applying a new T-mesh untangling and smoothing procedure. Bivariate T-spline representation is calculated by imposing the interpolation conditions, where the nodes of the T-mesh are used as interpolation points. The efficacy of the proposed technique is shown in several examples.

References

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