

Smoothing of Surface Triangulations Using Optimal Local Projection

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ABSTRACT

In environmental modeling (air pollution, wind field [1], etc.) that occurs in three-dimensional domains defined over complex terrains, a mesh generator capable of adapting itself to topographical data, to chimney geometry and to the numerical solution is essential. For this purpose, in past works we have developed an automatic unstructured tetrahedral mesh generator [2], by using a refinement/derefinement algorithm for two-dimensional domains and a version of 3-D Delaunay triangulation. Occasionally in this process, low quality or even inverted elements may appear. For this reason, we also introduced a simultaneous untangling and smoothing procedure to optimize the resulting meshes [3]. Besides, a new problem arose with the quality of surface triangulation of irregular terrains, since, in order to prevent a loss of details of the original surface information, we did not allow the movement of nodes placed over the terrain. This problem motivates the introduction in this paper of a new procedure to improve the quality of triangular meshes defined on surfaces. The improvement is obtained by an iterative process in which each node of the mesh is moved to a new position that minimizes certain objective function. This objective function is derived from an algebraic quality measures of the local mesh [4,5] (the set of triangles connected to the adjustable or *free node*). If we allow the free node to move on the surface without imposing any restriction, only guided by the improvement of the quality, the optimization procedure can construct a high-quality local mesh, but with this node in an *unacceptable* position. To avoid this problem the optimization is done in the *parametric mesh*, where the presence of barriers in the objective function maintains the free node inside the *feasible region*. In this way, the original problem on the surface is transformed into a two-dimensional one on the *parametric space*. In our case, the parametric space is an optimal plane, chosen in terms of the local mesh, in such a way that this mesh can be projected performing a *valid* mesh, that is, without *inverted* elements. Several examples and applications presented in this work show how this technique is capable to improve the quality of surface meshes maintaining their topologies.

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