Incomplete factorization for preconditioning shifted linear systems

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The resolution of several problems of science and engineering, such as parabolic partial differential equations, mass consistent models for wind field adjustment [1, 2], etc., with any discretization technique, yields linear systems of equations of the form,

$$(M + \varepsilon N)\, x_\varepsilon = b_\varepsilon \tag{1}$$

where M and N are constant for a given discretization. In these problems, the system (1) must be solved for different values of ε.
Iterative solvers based on Krylov subspaces are the most efficient methods for such large and sparse linear systems [3]. In our case, since M and N are symmetric positive definite matrices, the Conjugate Gradient (CG) provides the best results. In addition, the use of suitable preconditioning techniques [4] allows a faster convergence of CG.

For preconditioning these systems, we can build a different preconditioner for each value of ε. In general, this means to obtain good convergence behaviour but at a high computational cost related to each preconditioner. On the contrary, we can use a unique preconditioner, the first of the above list, for solving all the linear systems. However, this second strategy may lead to convergences as slow as the value of ε is far from the initial value ε_0 chosen for building the preconditioner.

In this work, an intermediate procedure is proposed. It consists of a preconditioner based on an incomplete Cholesky factorization that may be updated for each new system at a low computational cost. Thus, it provides better convergence than the latter strategy and is cheaper that the former. In a similar way, Meurant [5] proposes this preconditioner for the special case $(M + \varepsilon D)\, x_\varepsilon = b_\varepsilon$, with D being a diagonal matrix. In addition, Benzi [6] develops a preconditioner, based on a factorized approximate inverse [7], for shifted linear systems of the form $(M + \varepsilon I)\, x_\varepsilon = b_\varepsilon$, with I being the unit matrix. This preconditioner may be updated in function of ε.

Several numerical experiments are presented in order to show the efficiency of the proposed preconditioner.

References

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