Optimization of surface meshes by projections on the plane. Applications to environmental problems.

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In this work we present a procedure to smooth meshes defined on surfaces. The construction of the objective function is done in the framework of theory of *algebraic quality measures* developed in [1]. For 2D or 3D meshes the quality improvement is obtained by an iterative process in which each node of the mesh is moved to a new position that minimizes certain objective function [2]. This function is derived from a quality measure of the *local mesh*, that is, the set of tetrahedra connected to the adjustable or *free node*. We have chosen, as a starting point, a 2D objective function that presents a barrier in the boundary of the *feasible region* (set of points where the free node could be placed to get a valid local mesh, that is, without *inverted elements*). This barrier has an important role because it avoids the optimization algorithm to create a tangled mesh when it starts with a valid one.

Nevertheless, objective functions constructed by algebraic quality measures are only directly applicable to 2D or 3D meshes, but they are not in surface meshes. To overcome this problem, the local mesh, $M(p)$, sited on a surface Σ, is orthogonally projected on a plane P (if this exists) in such a way that it performs a valid local mesh $N(q)$. Here p is the free node on Σ and q is its projection on P. The optimization of $M(p)$ is got by the appropriated optimization of $N(q)$. To do this we search *ideal* triangles in $N(q)$ that become equilateral ones in $M(p)$.

In general, when the local mesh $M(p)$ is on a curved surface, each triangle is placed on a different plane and it is not possible to define a feasible region. Indeed, it is not clear the concept of valid mesh in this case. In order to make sense of it, we say that $M(p)$ is *acceptable* if $N(q)$ is valid. Note that the feasible region is always perfectly defined in $N(q)$.

To construct the objective function in $N(q)$, it is first necessary to define the objective function in $M(p)$ and, afterwards, to establish the connection between them. A crucial aspect for this construction is to keep the barrier of the 2D objective function. This is got by doing a suitable approximation in the process that transforms the original problem on Σ into an entirely two-dimensional one.

Several examples and applications presented in this work show how this technique is capable to improve the quality of surface meshes. In particular, the algorithm is specially easy to implement when the plane P is the same for all the mesh. This is the case of meshes adapted to the terrain used for the simulation of environmental problems [3].

References

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