A New Predictive Solar Radiation Numerical Model

F. Díaz*, G. Montero, J.M. Escobar, E. Rodríguez, R. Montenegro

University Institute for Intelligent Systems and Numerical Applications in Engineering, University of Las Palmas de Gran Canaria, Edificio Central del Parque Tecnológico, Campus de Tafira, 35017, Las Palmas de Gran Canaria, Spain

Abstract

A solar radiation numerical model is presented. With it, the user can estimate the radiation values in any location easily and compute the solar power generation taking into account not only the radiation level, but also the terrain surface conditions considering the cast shadows. The terrain surface is taken into account, using 2-D adaptive meshes of triangles which are constructed using a refinement/derefinement procedure in accordance with the variations of terrain surface and albedo. The model can be used in atmospheric sciences as well as in electrical engineering since it allows the user to find the optimal location for the maximum power generation in photovoltaic or solar thermal power exploitaitions. For this purpose, the effect of shadows is considered in each time step. Solar radiation is first computed for clear-sky conditions and then, real-sky values are computed daily in terms of the clear-sky index. Maps for clear-sky index are obtained from a spatial interpolation of observational data which are available for each day at several points of the studied zone. Finally, the solar radiation maps of a month are calculated from the daily results. However, for power system management purposes, it is very important to know the amount of energy that a facility can introduce into the grid in a future. That is why a predictive tool has been developed. So, the model can be applied in solar radiation forecasting using a meteorological model. The estimation of daily solar radiation provided by such model is used to adjust the clear sky results and, then, to obtain the real sky radiation.

Keywords: solar radiation, adaptive meshes, solar power, forecasting

*Corresponding author
Email address: felipe.diaz@ulpgc.es (F. Díaz)

1. Introduction

In the last years, the impact of the different renewable energies in the power systems has greatly increased. Considering [1], the world total renewable power capacity for 2013, not including hydro, is over 560 GW. The most important source among them is the wind power (318 GW) followed by the solar PV capacity (139 GW). On the other hand, we have 3.4 GW in concentrating solar thermal power[1]. Figure 1 shows the evolution of the global PV cumulative installed capacity in the world. As can be seen, the trend is exponential. The results of [2] let us learn that that PV penetration in Europe in 2030 could be between 10% and 15% of electricity demand.

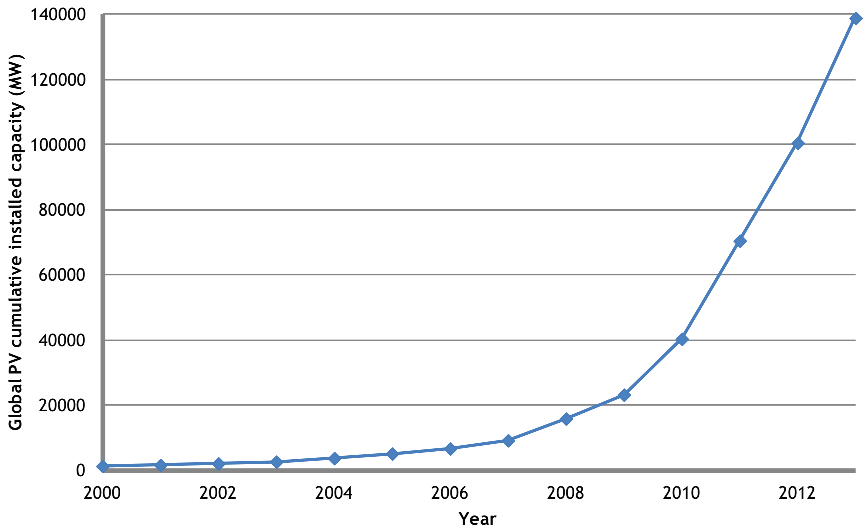

Figure 1: Global PV cumulative installed capacity (MW)

So power systems will be, each time, more and more influenced by solar power facilities. To reach high penetration, this renewable source needs to become a manageable one and that is the reason why, from the power systems point of view, is so important the research in solar radiation forecasting: If

[1] Practically all this capacity is installed in two countries: Spain and the USA

we need to know the actual generation capacity for tomorrow, we will need to know the radiation values for that day.

Starting from the spatial model developed by Šúri and Hofierka [3], which is based on astrophysical, atmospheric, physical and geometrical considerations, the calculation of solar radiation on the terrain [4, 5] is studied. The factors related to the terrain such as the elevation, the albedo, the surface inclination or the cast shadows, are essential to have a precise idea of how the radiation comes into contact with the surface. An adaptive mesh of triangles [4, 5] to represent the terrain and its actual orography as a solid surface which really casts shadows, is used. Mesh refinement/derefinement techniques have been widely used in other scientific problems [6, 7].

2. Modelling the influence of terrain characteristics

To model the terrain characteristics, an adaptive procedure of mesh refinement/derefinement [4] has been carried out using two different derefinement parameters, one for the orography and one for albedo. Although the problem requires the use of four variables, namely, the nodes coordinates, height and albedo, in reality the construction of the mesh is a two-dimensional problem.

Starting from a regular triangulation τ_1 of the rectangular region under study, a sequence of nested meshes $\Gamma = \{\tau_1 < \tau_2 < \ldots < \tau_m\}$ will be built. The triangulation is such that level τ_j is obtained by a global refinement of the previous level τ_{j-1} with the 4-T Rivara's algorithm [8]. The number of levels m of the sequence is determined by the degree of discretization of the terrain[2], so we can ensure that this regular mesh is able to capture all the orographic and albedo information by an interpolation of the heights and albedo in the nodes of the mesh.

Once this is done, we have to define a new sequence $\Gamma' = \{\tau_1 < \tau_2' < \ldots < \tau_{m'}'\}, m' \leq m$, by applying a derefinement algorithm [7]. By means of two derefinement parameters, ε_h and ε_a, we can determine the accuracy of the approximation to terrain surface and albedo.

The accurate estimation of the solar radiation on a terrain surface needs to take into account the cast shadows. This problem is, after all, a geometrical one: a triangle will be shadowed when, looking at the mesh from the sun, we can find another triangle that totally or partially covers the former.

[2]Elevation Map

One way to face this issue is constructing a reference system x', y' and z', with z' in the direction of the beam radiation (see Figure 2). After this is done the mesh needs to be projected on the plane $x'y'$ to see, for each triangle, if there are any other triangles overlapped.

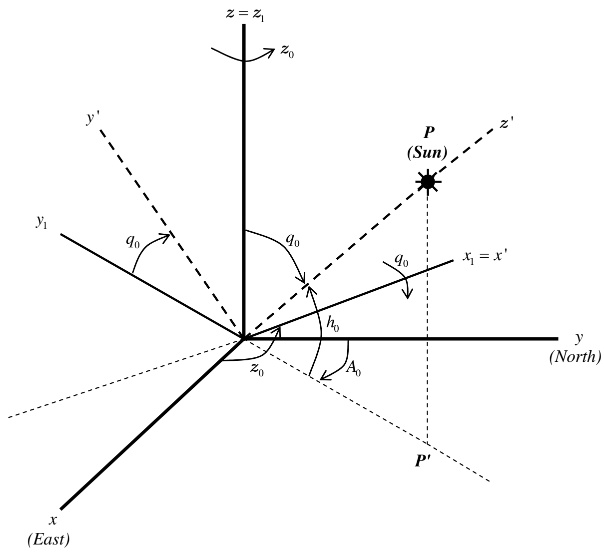

Figure 2: Reference systems and Euler angles

The sun position with respect to a horizontal surface is given by two coordinates, the solar altitude h_0 and the solar azimuth A_0 (see Figure 2). As said above, we need to make a coordinate transformation to get z' in the direction of the beam radiation. Thus, we need to know a vector that defines the solar beam direction for any time and position. In the literature we can find many ways to reach this. Blanco-Muriel et al. proposed the so called PSA algorithm [9], developed at the *Plataforma Solar de Almería*.

Once the coordinates transformation is done, the intersection between triangles is checked.

We assign a different level of lighting or shade to each triangle of the mesh depending on the number of warning points that lie inside other triangles (see Figure 3). This factor, L_f, is used in the estimation of beam and diffuse irradiances.

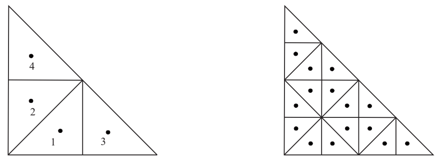

Figure 3: Warning points for shading analysis

3. Clear sky solar radiation modelling

Once the terrain is discretized by means of adaptive meshes, the radiation values on every triangle will be computed. A *solar radiation model* based on the works of Šúri and Hofierka [3] is developed, taking into account the shadows over each triangle of the surface [4, 5] for each time step.

The results will be the clear sky irradiance values for each triangle. The global solar irradiance comprises three different components: *beam*, *diffuse* and *reflected* irradiances. The first component, though partially attenuated by the atmosphere, is not reflected or scattered and reaches the surface directly. The second one is the scattered irradiance that reaches the terrain surface and that goes in all directions and produces no shadows because of the inserted opaque objects. The third one is the irradiance which is reflected from the surface onto an inclined receiver and depends on the ground albedo.

3.1. Beam radiation

According to J.K. Page [10], we will take the solar constant outside the atmosphere at the mean solar distance, I_0, as 1367 (W/m^2). Due to the earth's orbit eccentricity, a correction factor ϵ is needed for the calculation of the extraterrestrial irradiance G_0.

$$G_0 = I_0 \epsilon \tag{1}$$

where ϵ depends on the day angle. The beam irradiance, normal to the solar beam, G_{b0} (W/m^2) is attenuated by the cloudless atmosphere, and calculated as follows:

$$G_{b0c} = G_0 \; exp\{-0.8662 T_{LK} m \delta_R(m)\} \tag{2}$$

The term $0.8662T_{LK}$ is the dimensionless Linke atmospheric turbidity factor corrected by F. Kasten [11]. Subindex c shows that we are calculating clear sky irradiances. The parameter m in (2) is the relative optical air mass calculated as proposed in [12], and $\delta_R(m)$ is the Rayleigh optical thickness at air mass m.

Taking into account what written above, the beam irradiance on a horizontal surface for clear sky conditions $G_{bc}(0)$ becomes,

$$G_{bc}(0) = G_{b0c}\, L_f\, \sin h_0 \tag{3}$$

where h_0 is the solar altitude angle and L_f, the lighting factor that corrects the beam irradiance as the surface is sunlit or shadowed. The beam irradiance on an inclined surface for clear sky conditions $G_{bc}(\beta)$ is obtained as,

$$G_{bc}(\beta) = G_{b0c} L_f \sin \delta_{exp} \tag{4}$$

where β is the angle between the inclined surface and the horizontal, and δ_{exp} is the solar incidence angle measured between the sun beam direction and its projection on an inclined surface.

3.2. Diffuse radiation

The estimation of the diffuse component in horizontal surfaces $G_{dc}(0)(\mathrm{W/m^2})$ is carried out using the equation,

$$G_{dc}(0) = G_0 T_n(T_{LK}) F_d(h_0) \tag{5}$$

As can be seen, $G_{dc}(0)$ is a function of the diffuse transmission T_n which, at the same time, depends on the Linke turbidity factor T_{LK}. Also, we have the function F_d which depends on the solar altitude h_0 [13]. To obtain the diffuse irradiance on a inclined surface, $G_{dc}(\gamma_N)$, being γ_N the angle between the normal to a triangle and the horizontal plane, both, sunlit and shadowed surfaces have to be considered following the equations proposed in [14].

3.3. Reflected radiation

The last component to take into account is the ground reflected irradiance under clear sky conditions ($G_r(\gamma_N)$). According to Muneer [15], this one will be proportional to the global horizontal irradiance $G_c(0)$, to the mean ground albedo ρ_g and a fraction of the ground viewed by an inclined surface $r_g(\gamma_N)$.

$$G_r(\gamma_N) = \rho_g G_c(0) r_g(\gamma_N) \tag{6}$$

where

$$\begin{aligned} r_g(\gamma_N) &= (1 - \cos\gamma_N)/2 & (7) \\ G_c(0) &= G_{bc}(0) + G_{dc}(0) & (8) \end{aligned}$$

4. Predicted real sky solar radiation

In the model developed by F. Díaz et al. [5], the real sky radiation values $G(0)$ were calculated as a correction of those for clear sky, $G_c(0)$, using the clear sky index k_c, known for all the measurement stations.

$$G(0) = G_c(0)k_c \tag{9}$$

These irradiance or irradiation values under real sky conditions are, for every time step along the typical year, the expected radiation values according to the climatic characteristics of the studied area. However, a step forward is needed. The correct management of power systems with high solar power influence needs more reliable irradiance values for the short and medium term.

As the weather conditions, especially cloudiness, will greatly influence the radiation values, we will need to have different clear sky indexes, not only for every triangle of the mesh, but for each and every time step too. This way, a new computation for every time step is needed. The problem of the forecasting can be addressed in different ways. Mainly we can find two basic approaches [16, 17],

1. Using numerical weather prediction (NWP) models
2. Projecting observed solar radiation conditions based on immediate measured history

In this case, our approach will be the first one. The weather forecasting model used can be any of the available ones, MM5, HIRLAM etc. The simulations presented in this work are done using the PSU/NCAR[3] mesoscale fifth generation model (MM5). This way, good medium[4] term predictions are obtained. MM5 estimates the irradiances on a mesh whose horizontal projection is itself a non-uniform grid, so it is necessary to interpolate these results into the centers of the adaptive mesh triangles.

The key issue, therefore, is the assessment of the clear sky index for all the triangles in our adaptive mesh. In this case, as the meteorological model provides results with a certain time step, we can find a clear sky index, k_{c_i}, for each one of those moments (see equation (10)). Note that the clear sky

[3]Pennsylvania State University / National Center for Atmospheric Research, USA
[4]Up to six days

radiation is used without shadows[5], as the meteorological model does not take them into account. Obviously, later the full shadows clear sky radiation will be used to get the final predicted real sky radiation. Also, the MM5 grid does not take into account the actual terrain characteristics and topography.

$$k_{c_i} = \frac{G(0)}{G_{c_{ss}}(0)} \tag{10}$$

The process to calculate the clear sky index will be:

1. To triangulate the MM5 grid using a flat triangulation in which the nodes are those from the MM5 mesh. Each node is assigned the predicted radiation in every time step
2. The MM5 triangulated mesh overlaps the 2D topography mesh. Each triangle belonging to the topography mesh, is assigned the interpolated irradiance (see Figure 4), using equation (11)

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} x_0 \\ y_0 \\ z_0 \end{bmatrix} + K_1 \cdot \begin{bmatrix} V_{1_x} \\ V_{1_y} \\ V_{1_z} \end{bmatrix} + K_2 \cdot \begin{bmatrix} V_{2_x} \\ V_{2_y} \\ V_{2_z} \end{bmatrix} \tag{11}$$

   Three equations in three unknowns, K_1, K_2 and z, that is the predicted irradiance for the adaptive mesh triangle.
3. The clear sky index is computed for each triangle and time step, using the previously computed self shadows clear sky values

As k_{c_i} values are known for all the triangles related to the MM5 grid, the next step will be the interpolation of the index for the whole zone. A simple formula that has also been used in other environmental problems defined on complex orography [18] is applied,

$$k_{c_i} = \varepsilon \frac{\sum_{n=1}^{N} \frac{k_{cn}}{d_n^2}}{\sum_{n=1}^{N} \frac{1}{d_n^2}} + (1-\varepsilon) \frac{\sum_{n=1}^{N} \frac{k_{cn}}{|\Delta h_n|}}{\sum_{n=1}^{N} \frac{1}{|\Delta h_n|}} \tag{12}$$

where k_{c_i} corresponds to the clear sky index at each point of the mesh for each time step, k_{cn} is the clear sky index obtained at the MM5 grid, N is the number of nodes in the MM5 grid used in the interpolation, d_n is the

[5] Subindex ss means self shadows

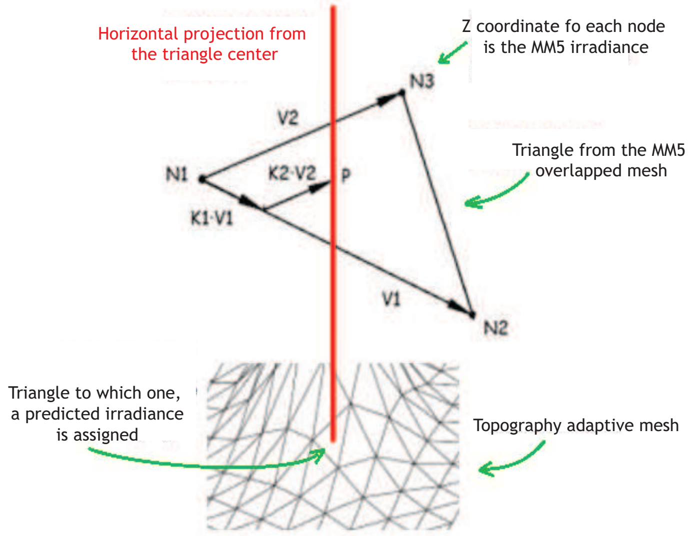

Figure 4: Predicted irradiance assignation

horizontal distance and $|\Delta h_n|$ is the difference in height between the node n and the studied point, respectively, and ε is a parameter between 0 and 1. Where the studied point coincides with a measurement station, equation (12) is not continuous. The continuity is ensured if we assume the predicted values, though considering the shadows, at those points.

In Figure 5 can be seen the structure of the process followed to get the predicted solar radiation values.

5. Simulation results

Simulations have been performed on the island of Gran Canaria (Canary Islands - Spain), which is located in the Atlantic Ocean in front of the north-west coast of Africa. The climatic irradiation values were studied in [5, 19]. In Figure 6, the adaptive mesh for Gran Canaria Island can be seen.

The studied case consists of a solar irradiance prediction carried out for Gran Canaria Island, for an episode on December, 23, 2009. As an example, two locations are presented: *Temisas*[6] and *Lomo Carbonero*[7]. As was said

[6] 27.917°N 15.491°W, number of triangle 3302, 2^{nd} quadrant of the island
[7] 28.033°N 15.533°W, number of triangle 9545, 1^{st} quadrant of the island

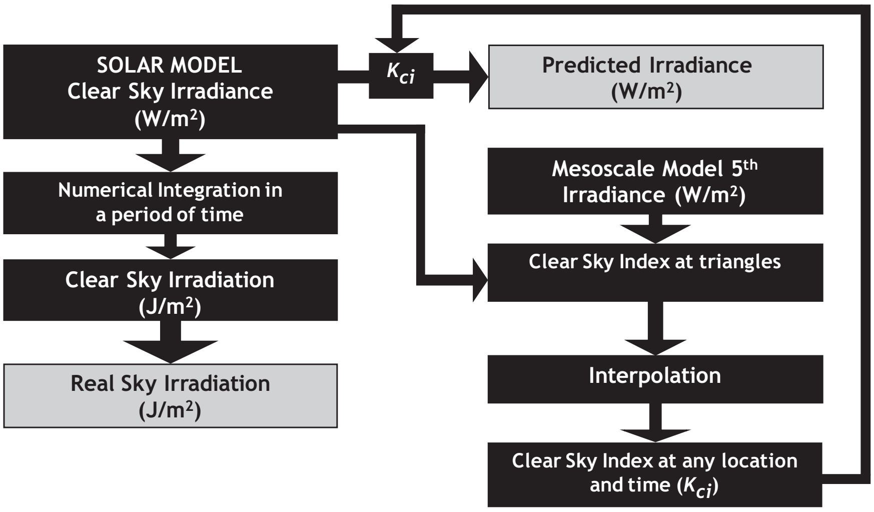

Figure 5: Predictive solar model outline

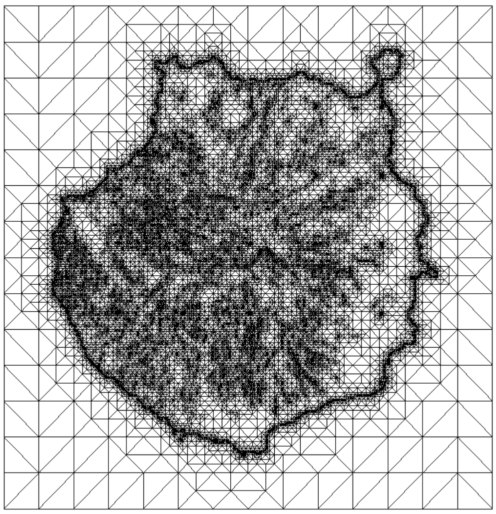

Figure 6: Adaptive mesh for Gran Canaria Island

above, given the real time variability of cloudiness, the clear sky index, k_{c_i}, is calculated for each triangle of the mesh, and for every time step. Thus, the predicted evolution of the cloudiness in the point of analysis is available.

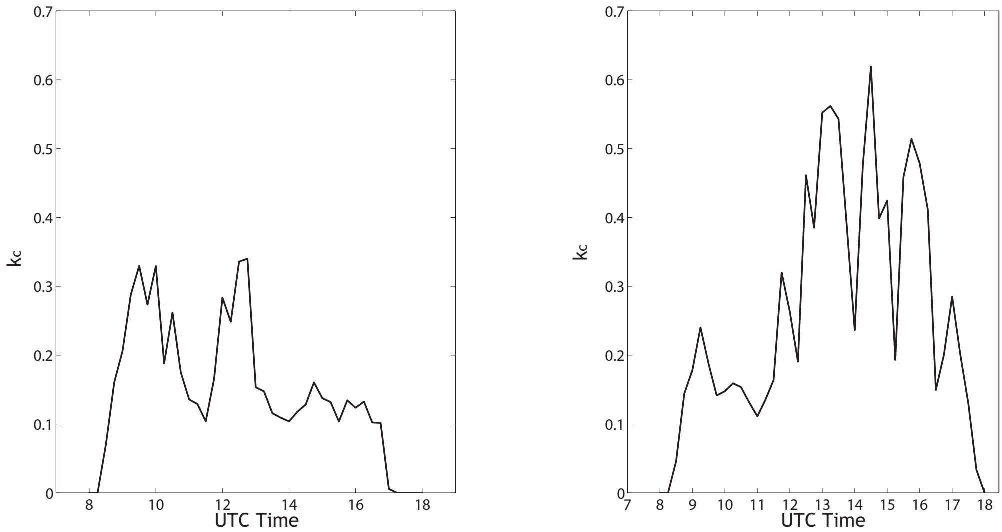

Figure 7: *Temisas* and *Lomo Carbonero* clear sky index. December, 23, 2009

Using this clear sky index distribution along the day, the predicted irradiance curve (predicted real sky), can be obtained for the considered episode. Calculations were performed for a horizontal surface.

In Figure 8 the predicted irradiance for 2009, December, 23^{rd} can be seen. In *Temisas* that one was, as deduced from the meteorological forecasting, a very cloudy day, getting apart from the expected typical climatic day.

However, in *Lomo Carbonero*, the forecasted day is less cloudy, and it is closer to the climatic typical day for that date. Remember that clear sky irradiance is the one computed without the presence of cloudiness and the real sky irradiance is the one which is expected to be measured in the reality.

Once we have the predicted irradiance, with the help of a Solar PV or Thermal Model, the predicted electric power generation can be obtained for the short and medium term. This allows the Power System Operator to develop a correct management (see Figure 9).

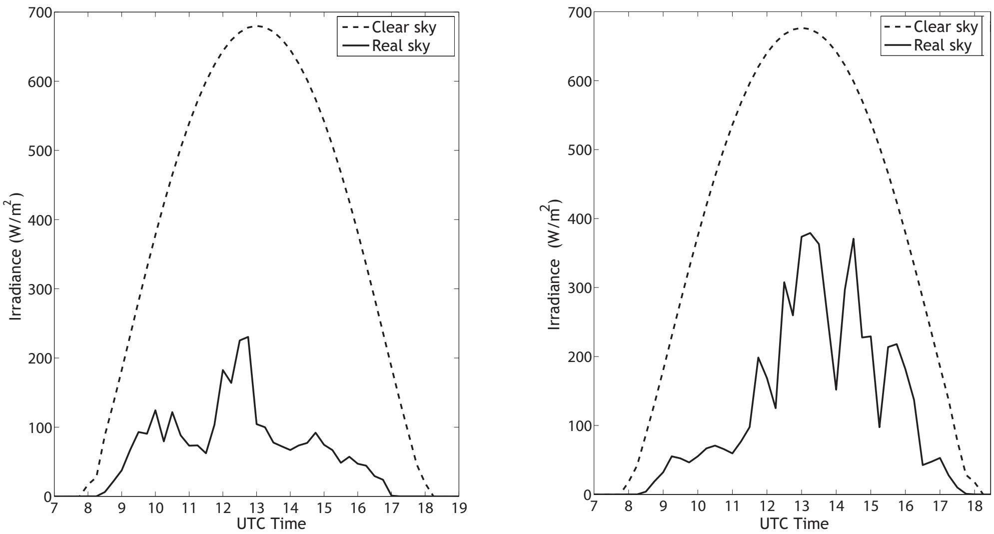

Figure 8: *Temisas* and *Lomo Carbonero* predicted irradiance for December, 23, 2009

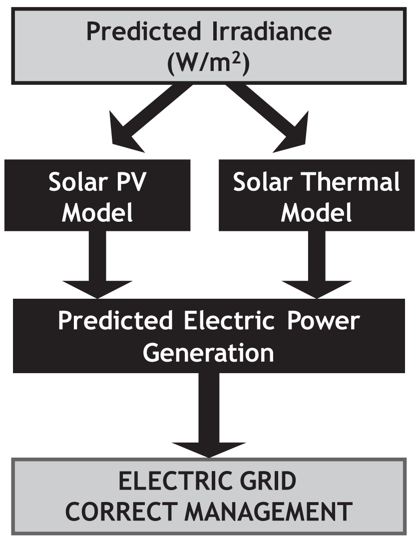

Figure 9: Predictive power model outline

6. Conclusions

A predictive numerical model for forecasting the solar radiation on a surface is presented. Needed requirements are the location, topography, albedo and weather forecasting data. Solar radiation on a surface is estimated taking into account the shadow distribution in each time step. For this purpose, the adaptivity of the triangulation related to the topography and albedo is essential. Adaptive meshes lead to a minimum computational cost, since the number of triangles to be used is optimum [5].

This predictive adaptive model gives us realistic values about the solar radiation in the short term and, especially, in the medium term with the accuracy of the weather forecasting model. As an example, some predictions have been done using the MM5 model.

As a result, the use of this predictive adaptive model allows to make a better management of the Power System due to the accurate knowledge of the electric power generated in the different solar facilities. For this aim, a solar photovoltaic model and/or a solar thermal model needs to be implemented to receive the results of the predictive model as an input. Moreover, rectangular collectors can be included in the model as composed by two triangles in the same plane, with a given inclination and orientation.

As a future research in this regard, it would be interesting to develop a predictive solar code based on artificial neural networks and compare results with this one.

7. Acknowledgments

This work has been supported by the *Dirección General de Investigación*, *Ministerio de Economía y Competitividad*, Spanish Government, grant contract CGL2011-29396-C03-01.

References

[1] Renewable Energy Policy Network for the 21st Century, "Renewables 2014. Global status report", Technical Report, Renewable Energy Policy Network for the 21st Century, 2014.

[2] European Photovoltaic Industry Association, "Global Market Outlook For Photovoltaics 2014-2018", Technical Report, European Photovoltaic Industry Association, 2014.

[3] M. Šúri, J. Hofierka, "A New GIS-based Solar Radiation Model and its application to photovoltaic assessments", Transactions in GIS 8 (2), 175–170. 2004.

[4] G. Montero, J.M. Escobar, E. Rodríguez, R. Montenegro, "Solar radiation and shadow modelling with adaptive triangular meshes", Solar Energy 83 (7), 998–1012, 2009.

[5] F. Díaz, G. Montero, J.M. Escobar, E. Rodríguez, R. Montenegro, "An Adaptive Solar Radiation Numerical Model", Journal of Computational and Applied Mathematics 236 (18), 4611–4622, 2012.

[6] G. Winter, G. Montero, L. Ferragut, R. Montenegro, "Adaptive strategies using standard and mixed finite element for wind field adjustment", Solar Energy 54 (1), 46–56, 1995.

[7] L. Ferragut, R. Montenegro, A. Plaza, "Efficient refinement/derefinement algorithm of nested meshes to solve evolution problems", Comm Num Meth Engrg 10, 403–412, 1994.

[8] M.C. Rivara, "A Grid Generator Based on 4-Triangles Conforming. Mesh-Refinement Algorithms", Int J Num Meth Engrg 24, 1343–1354, 1987.

[9] M. Blanco-Muriel, D.C. Alarcón-Padilla, T. López-Moratalla, M. Lara-Coira, "Computing the solar vector", Solar Energy 70 (5), 431–441, 2001.

[10] J.K. Page (ed), "Prediction of Solar Radiation on Inclined Surfaces", D. Reidel Publishing Co., Dordrecht, 1986.

[11] F. Kasten, "The Linke turbidity factor based on improved values of the integral Rayleigh optical thickness", Solar Energy 56 (3), 239–244, 1996

[12] F. Kasten, A.T. Young, "Revised optical air mass tables and approximation formula", Applied Optics 28, 4735–4738, 1989.

[13] K. Scharmer, J. Greif, "The European Solar Radiation Atlas.Vol. 2 : Database and exploitation software", Les Presses de l'École des Mines, Paris, 2000.

[14] T. Muneer, "Solar Radiation model for Europe", Building Services Engineering Research and Technology 11, 153–163, 1990.

[15] T. Muneer, "Solar Radiation and Daylight Models for Energy Efficient Design of Buildings", Architectural Press, Oxford, 1997.

[16] R. Perez, S. Kivalov, J. Schlemmer, K. Hemker, D. Renn, T.E. Hoff, "Validation of short and medium term operational solar radiation forecasts in the US", Solar Energy 84 (12), 2161–2172, 2010.

[17] R. Perez, K. Moore, S. Wilcox, D. Renn, A. Zelenka, "Forecasting solar radiation Preliminary evaluation of an approach based upon the national forecast database", Solar Energy 81 (6), 809–812, 2007.

[18] G. Montero, R. Montenegro, J.M. Escobar, "A 3-D Diagnostic Model for Wind Field Adjustment", J Wind Engrg Ind Aer 74-76, 249–7261, 1998.

[19] F. Díaz, G. Montero, J.M. Escobar, E. Rodríguez, R. Montenegro , "Solar radiation estimation using a numerical model", in F.O. Hocaoglu (ed.) *Solar radiation: protection, management and measurement techniques*, Nova Publishers, New York, 2012.