Contributions to Forest Fire Simulations: Mathematical Models, Numerical Methods and GIS Integration

L. Ferragut, M. Asensio, S.Monedero†

IUFFyM Instituto Universitario de Física Fundamental y Matemáticas
Numerical Simulation and Scientific Computation Group
University of Salamanca, Spain
† Tecnosylva SL, Parque Tecnológico de León, Spain

April 16, 2008

Clasification

Models

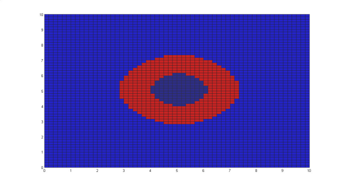
PROPAGATION	COMBUSTION
Position of the fire front	Modelization of physics

models with increasing complexity:

- Cellular Automata
- Geometric
- Empiric models
- Reaction Diffusion Convection

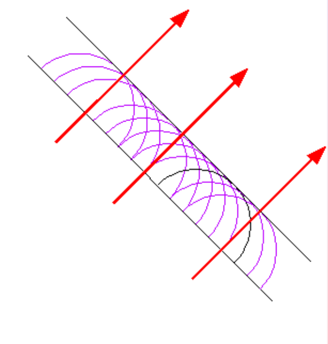
Cellular Automata

- Cell with several states (burning, burnt, not burning)
- Transition probabilities as a function of the neigbhours cells
  - Advantages: $\Longrightarrow$ Fast computation
  - Disadvantages:
    It modelize probabilistic phenomena
    Not direct relation with physical parameters



Geometric models

- 1D Fire front on a 2D surface
- Huygens principle
- Advantages and disadvantages the same as $\Longrightarrow$ cellular automata.



Empiric models

- An energy balance is considered on the fire front
- Rate of spread given by empiric laws
- R.C.Rothermel,
  A Mathematical Model for Predicting Fire Spread in Wildland fuels.
- Advantages $\Longrightarrow$ Fast computation
- Disadvantages $\Longrightarrow$ Parameters must be adjusted case by case.

Combustion models: Convection-Difussion-Radiation-Reaction

Complex models

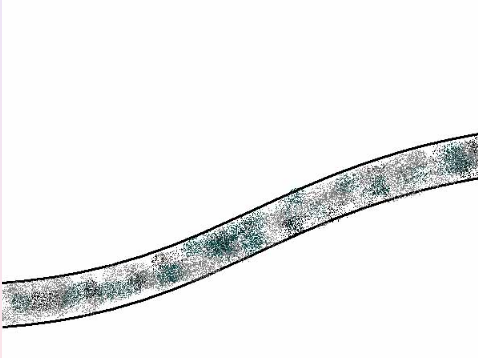
- Several phases and conservation laws are considered
- Solid phase and gas phase with different temperature
- Two layer models
- Three dimensional equations
- Large time computation

Simplified models

- An average medium is considered
- Only one temperature
- One phase (other phases parametrized)
- One or two dimensional equations considered
- Could allow real time computation or faster

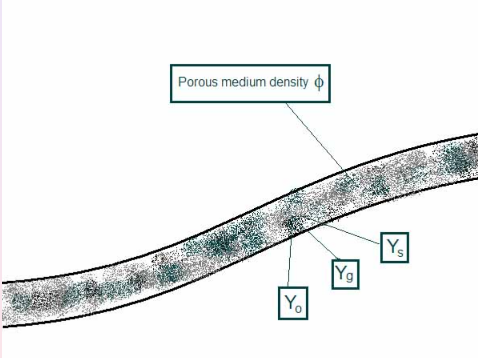
Combustion models: A general example. Physical Assumptions

1. Porous medium of porosity ϕ and density ρ
2. Composed of solid fuel Y_s, oxygen Y_o and gaseous fuel Y_g
3. Gases are all assumed to have an equal temperature T_g. The solid temperature T_s is coupled by the term $h(T_s - T_g)$
4. Solid fuel Y_s transforms into gaseous fuel Y_g through pyrolysis.
5. Gaseous fuel Y_g reacts with the oxygen Y_o
6. Which generates the flames and heat
7. Gases and temperatures are under the influence of convection, diffusion and heat is loss in the vertical direction (cooling)
8. Gain in solid temperature due to radiation R and combustion heat (proportional to the solid fuel combusted)



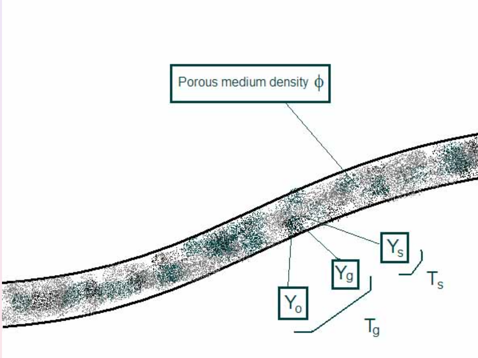
Combustion models: A general example. Physical Assumptions

1. Porous medium of porosity ϕ and density ρ
2. Composed of solid fuel Y_s, oxygen Y_o and gaseous fuel Y_g
3. Gases are all assumed to have an equal temperature T_g. The solid temperature T_s is coupled by the term $h(T_s - T_g)$
4. Solid fuel Y_s transforms into gaseous fuel Y_g through pyrolysis.
5. Gaseous fuel Y_g reacts with the oxygen Y_o
6. Which generates the flames and heat
7. Gases and temperatures are under the influence of convection, diffusion and heat is loss in the vertical direction (cooling)
8. Gain in solid temperature due to radiation R and combustion heat (proportional to the solid fuel combusted)



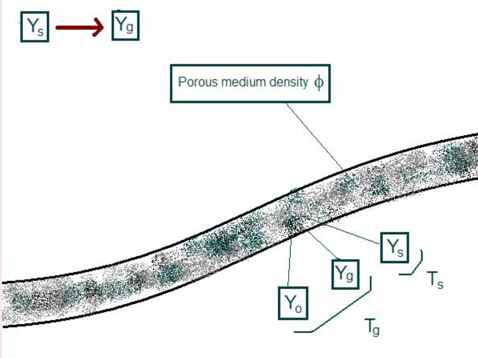
Combustion models: A general example. Physical Assumptions

1. Porous medium of porosity ϕ and density ρ
2. Composed of solid fuel Y_s, oxygen Y_o and gaseous fuel Y_g
3. Gases are all assumed to have an equal temperature T_g. The solid temperature T_s is coupled by the term $h(T_s - T_g)$
4. Solid fuel Y_s transforms into gaseous fuel Y_g through pyrolysis.
5. Gaseous fuel Y_g reacts with the oxygen Y_o
6. Which generates the flames and heat
7. Gases and temperatures are under the influence of convection, diffusion and heat is loss in the vertical direction (cooling)
8. Gain in solid temperature due to radiation R and combustion heat (proportional to the solid fuel combusted)



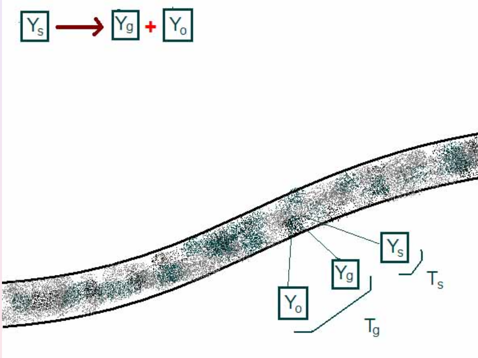
Combustion models: A general example. Physical Assumptions

1. Porous medium of porosity ϕ and density ρ
2. Composed of solid fuel Y_s, oxygen Y_o and gaseous fuel Y_g
3. Gases are all assumed to have an equal temperature T_g. The solid temperature T_s is coupled by the term $h(T_s - T_g)$
4. Solid fuel Y_s transforms into gaseous fuel Y_g through pyrolysis.
5. Gaseous fuel Y_g reacts with the oxygen Y_o
6. Which generates the flames and heat
7. Gases and temperatures are under the influence of convection, diffusion and heat is loss in the vertical direction (cooling)
8. Gain in solid temperature due to radiation R and combustion heat (proportional to the solid fuel combusted)



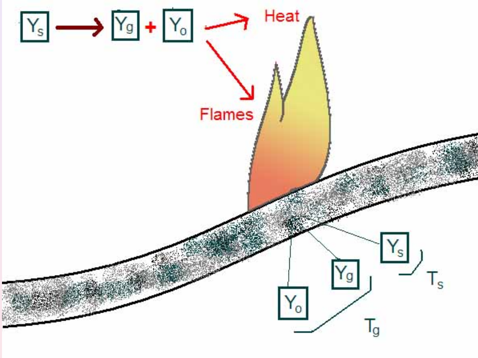
Combustion models: A general example. Physical Assumptions

1. Porous medium of porosity ϕ and density ρ
2. Composed of solid fuel Y_s, oxygen Y_o and gaseous fuel Y_g
3. Gases are all assumed to have an equal temperature T_g. The solid temperature T_s is coupled by the term $h(T_s - T_g)$
4. Solid fuel Y_s transforms into gaseous fuel Y_g through pyrolysis.
5. Gaseous fuel Y_g reacts with the oxygen Y_o
6. Which generates the flames and heat
7. Gases and temperatures are under the influence of convection, diffusion and heat is loss in the vertical direction (cooling)
8. Gain in solid temperature due to radiation R and combustion heat (proportional to the solid fuel combusted)



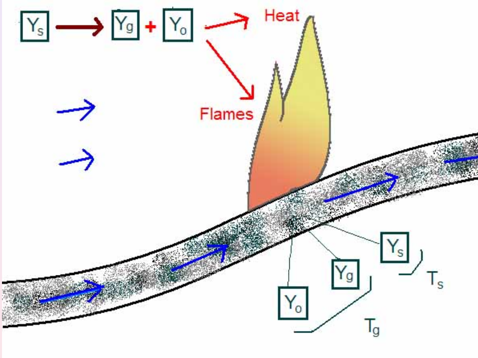
Combustion models: A general example. Physical Assumptions

1. Porous medium of porosity ϕ and density ρ
2. Composed of solid fuel Y_s, oxygen Y_o and gaseous fuel Y_g
3. Gases are all assumed to have an equal temperature T_g. The solid temperature T_s is coupled by the term $h(T_s - T_g)$
4. Solid fuel Y_s transforms into gaseous fuel Y_g through pyrolysis.
5. Gaseous fuel Y_g reacts with the oxygen Y_o
6. Which generates the flames and heat
7. Gases and temperatures are under the influence of convection, diffusion and heat is loss in the vertical direction (cooling)
8. Gain in solid temperature due to radiation R and combustion heat (proportional to the solid fuel combusted)



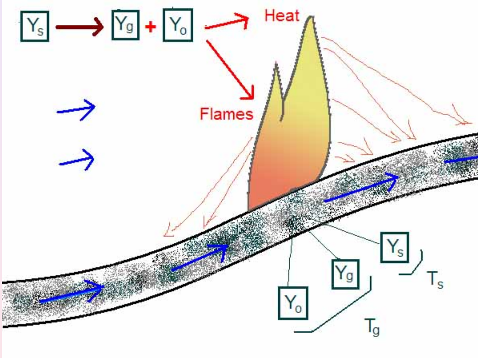
Combustion models: A general example. Physical Assumptions

1. Porous medium of porosity ϕ and density ρ
2. Composed of solid fuel Y_s, oxygen Y_o and gaseous fuel Y_g
3. Gases are all assumed to have an equal temperature T_g. The solid temperature T_s is coupled by the term $h(T_s - T_g)$
4. Solid fuel Y_s transforms into gaseous fuel Y_g through pyrolysis.
5. Gaseous fuel Y_g reacts with the oxygen Y_o
6. Which generates the flames and heat
7. Gases and temperatures are under the influence of convection, diffusion and heat is loss in the vertical direction (cooling)
8. Gain in solid temperature due to radiation R and combustion heat (proportional to the solid fuel combusted)



Combustion models: A general example. Physical Assumptions

1. Porous medium of porosity ϕ and density ρ
2. Composed of solid fuel Y_s, oxygen Y_o and gaseous fuel Y_g
3. Gases are all assumed to have an equal temperature T_g. The solid temperature T_s is coupled by the term $h(T_s - T_g)$
4. Solid fuel Y_s transforms into gaseous fuel Y_g through pyrolysis.
5. Gaseous fuel Y_g reacts with the oxygen Y_o
6. Which generates the flames and heat
7. Gases and temperatures are under the influence of convection, diffusion and heat is loss in the vertical direction (cooling)
8. Gain in solid temperature due to radiation R and combustion heat (proportional to the solid fuel combusted)



Combustion models: A general example. Equations

1. $(1-\phi)\rho_s C_s \frac{\partial T_s}{\partial t} - \nabla(\kappa_s \nabla T_s) = R(T_g) - h(T_s - T_g)$
2. $\phi\rho C(\frac{\partial T_g}{\partial t} + \mathbf{v}\cdot\nabla T_g) - \nabla(\kappa_g \nabla T_g) = -h(T_g - T_s) + qA_g Y_o Y_g e^{-\frac{E_a}{RT_g}} - h_s(T_g - T_\infty)$
3. $\frac{\partial Y_s}{\partial t} = -A_s Y_s e^{-\frac{E_a}{RT_s}}$
4. $\frac{\partial Y_o}{\partial t} + \mathbf{v}\cdot\nabla Y_o - \nabla(\kappa_{oo}\nabla Y_o) = -h_o(Y_o - Y_{o,\infty}) - A_g Y_o Y_g e^{-\frac{E_a}{RT_g}}$
5. $\frac{\partial Y_g}{\partial t} + \mathbf{v}\cdot\nabla Y_g - \nabla(\kappa_{gg}\nabla Y_g) = -h_g(Y_g - 0) - A_g Y_o Y_g e^{-\frac{E_a}{RT_g}} + AY_s e^{-\frac{E_a}{RT_s}}$

- T_s Temperature of the solid
- T_g Temperature of the gases
- Y_s Solid fuel
- Y_o Oxygen
- Y_g Gaseous fuel

Combustion models: A general example. Equations

1. $(1-\phi)\rho_s C_s \frac{\partial T_s}{\partial t} - \nabla(\kappa_s \nabla T_s) = R(T_g) - h(T_s - T_g)$
2. $\phi \rho C(\frac{\partial T_g}{\partial t} + \mathbf{v} \cdot \nabla T_g) - \nabla(\kappa_g \nabla T_g) = -h(T_g - T_s) + q A_g Y_o Y_g e^{-\frac{E_a}{RT_g}} - h_s(T_g - T_\infty)$
3. $\frac{\partial Y_s}{\partial t} = -A_s Y_s e^{-\frac{E_a}{RT_s}}$
4. $\frac{\partial Y_o}{\partial t} + \mathbf{v} \cdot \nabla Y_o - \nabla(\kappa_{oo} \nabla Y_o) = -h_o(Y_o - Y_{o,\infty}) - A_g Y_o Y_g e^{-\frac{E_a}{RT_g}}$
5. $\frac{\partial Y_g}{\partial t} + \mathbf{v} \cdot \nabla Y_g - \nabla(\kappa_{gg} \nabla Y_g) = -h_g(Y_g - 0) - A_g Y_o Y_g e^{-\frac{E_a}{RT_g}} + A Y_s e^{-\frac{E_a}{RT_s}}$

- T_s Temperature of the solid
- T_g Temperature of the gases
- Y_s Solid fuel
- Y_o Oxygen
- Y_g Gaseous fuel

Combustion models: A general example. Equations

1. $(1-\phi)\rho_s C_s \frac{\partial T_s}{\partial t} - \nabla(\kappa_s \nabla T_s) = R(T_g) - h(T_s - T_g)$
2. $\phi \rho C(\frac{\partial T_g}{\partial t} + \mathbf{v}\cdot\nabla T_g) - \nabla(\kappa_g \nabla T_g) = -h(T_g - T_s) + qA_g Y_o Y_g e^{-\frac{E_a}{RT_g}} - h_s(T_g - T_\infty)$
3. $\frac{\partial Y_s}{\partial t} = -A_s Y_s e^{-\frac{E_a}{RT_s}}$
4. $\frac{\partial Y_o}{\partial t} + \mathbf{v}\cdot\nabla Y_o - \nabla(\kappa_{oo}\nabla Y_o) = -h_o(Y_o - Y_{o,\infty}) - A_g Y_o Y_g e^{-\frac{E_a}{RT_g}}$
5. $\frac{\partial Y_g}{\partial t} + \mathbf{v}\cdot\nabla Y_g - \nabla(\kappa_{gg}\nabla Y_g) = -h_g(Y_g - 0) - A_g Y_o Y_g e^{-\frac{E_a}{RT_g}} + AY_s e^{-\frac{E_a}{RT_s}}$

- T_s Temperature of the solid
- T_g Temperature of the gases
- Y_s Solid fuel
- Y_o Oxygen
- Y_g Gaseous fuel

Combustion models: A general example. Equations

1. $(1-\phi)\rho_s C_s \frac{\partial T_s}{\partial t} - \nabla(\kappa_s \nabla T_s) = R(T_g) - h(T_s - T_g)$
2. $\phi \rho C(\frac{\partial T_g}{\partial t} + \mathbf{v}\cdot\nabla T_g) - \nabla(\kappa_g \nabla T_g) = -h(T_g - T_s) + qA_g Y_o Y_g e^{-\frac{E_a}{RT_g}} - h_s(T_g - T_\infty)$
3. $\frac{\partial Y_s}{\partial t} = -A_s Y_s e^{-\frac{E_a}{RT_s}}$
4. $\frac{\partial Y_o}{\partial t} + \mathbf{v}\cdot\nabla Y_o - \nabla(\kappa_{oo}\nabla Y_o) = -h_o(Y_o - Y_{o,\infty}) - A_g Y_o Y_g e^{-\frac{E_a}{RT_g}}$
5. $\frac{\partial Y_g}{\partial t} + \mathbf{v}\cdot\nabla Y_g - \nabla(\kappa_{gg}\nabla Y_g) = -h_g(Y_g - 0) - A_g Y_o Y_g e^{-\frac{E_a}{RT_g}} + A Y_s e^{-\frac{E_a}{RT_s}}$

- T_s Temperature of the solid
- T_g Temperature of the gases
- Y_s Solid fuel
- Y_o Oxygen
- Y_g Gaseous fuel

Combustion models: A general example. Equations

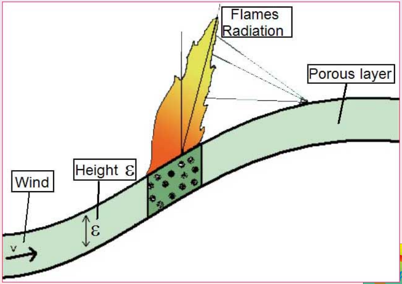
1. $(1-\phi)\rho_s C_s \frac{\partial T_s}{\partial t} - \nabla(\kappa_s \nabla T_s) = R(T_g) - h(T_s - T_g)$
2. $\phi\rho C(\frac{\partial T_g}{\partial t} + \mathbf{v}\cdot\nabla T_g) - \nabla(\kappa_g \nabla T_g) = -h(T_g - T_s) + qA_g Y_o Y_g e^{-\frac{E_a}{RT_g}} - h_s(T_g - T_\infty)$
3. $\frac{\partial Y_s}{\partial t} = -A_s Y_s e^{-\frac{E_a}{RT_s}}$
4. $\frac{\partial Y_o}{\partial t} + \mathbf{v}\cdot\nabla Y_o - \nabla(\kappa_{oo}\nabla Y_o) = -h_o(Y_o - Y_{o,\infty}) - A_g Y_o Y_g e^{-\frac{E_a}{RT_g}}$
5. $\frac{\partial Y_g}{\partial t} + \mathbf{v}\cdot\nabla Y_g - \nabla(\kappa_{gg}\nabla Y_g) = -h_g(Y_g - 0) - A_g Y_o Y_g e^{-\frac{E_a}{RT_g}} + AY_s e^{-\frac{E_a}{RT_s}}$

- T_s Temperature of the solid
- T_g Temperature of the gases
- Y_s Solid fuel
- Y_o Oxygen
- Y_g Gaseous fuel

Combustion models: A general example. Equations

1. $(1-\phi)\rho_s C_s \frac{\partial T_s}{\partial t} - \nabla(\kappa_s \nabla T_s) = R(T_g) - h(T_s - T_g)$
2. $\phi\rho C(\frac{\partial T_g}{\partial t} + \mathbf{v}\cdot\nabla T_g) - \nabla(\kappa_g \nabla T_g) = -h(T_g - T_s) + qA_g Y_o Y_g e^{-\frac{E_a}{RT_g}} - h_s(T_g - T_\infty)$
3. $\frac{\partial Y_s}{\partial t} = -A_s Y_s e^{-\frac{E_a}{RT_s}}$
4. $\frac{\partial Y_o}{\partial t} + \mathbf{v}\cdot\nabla Y_o - \nabla(\kappa_{oo}\nabla Y_o) = -h_o(Y_o - Y_{o,\infty}) - A_g Y_o Y_g e^{-\frac{E_a}{RT_g}}$
5. $\frac{\partial Y_g}{\partial t} + \mathbf{v}\cdot\nabla Y_g - \nabla(\kappa_{gg}\nabla T_s) = -h_g(Y_g - 0) - A_g Y_o Y_g e^{-\frac{E_a}{RT_g}} + AY_s e^{-\frac{E_a}{RT_s}}$

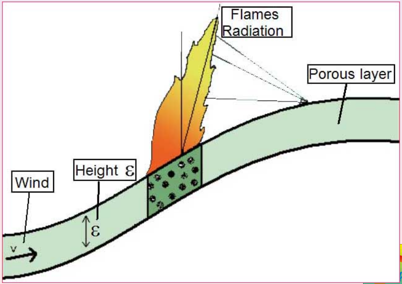
- Time terms
- Convective transport due to the wind
- Diffusion within the vegetation
- Vertical cooling or vertical loss
- Link between T_s and T_g
- Arrhenius type laws for combustion and pyrolysis
- Radiation term $R(T_g)$



Combustion models: A general example. Equations

1. $(1-\phi)\rho_s C_s \frac{\partial T_s}{\partial t} - \nabla(\kappa_s \nabla T_s) = R(T_g) - h(T_s - T_g)$
2. $\phi\rho C(\frac{\partial T_g}{\partial t} + \mathbf{v}\cdot\nabla T_g) - \nabla(\kappa_g \nabla T_g) = -h(T_g - T_s) + qA_g Y_o Y_g e^{-\frac{E_a}{RT_g}} - h_s(T_g - T_\infty)$
3. $\frac{\partial Y_s}{\partial t} = -A_s Y_s e^{-\frac{E_a}{RT_s}}$
4. $\frac{\partial Y_o}{\partial t} + \mathbf{v}\cdot\nabla Y_o - \nabla(\kappa_{oo}\nabla Y_o) = -h_o(Y_o - Y_{o,\infty}) - A_g Y_o Y_g e^{-\frac{E_a}{RT_g}}$
5. $\frac{\partial Y_g}{\partial t} + \mathbf{v}\cdot\nabla Y_g - \nabla(\kappa_{gg}\nabla T_s) = -h_g(Y_g - 0) - A_g Y_o Y_g e^{-\frac{E_a}{RT_g}} + AY_s e^{-\frac{E_a}{RT_s}}$

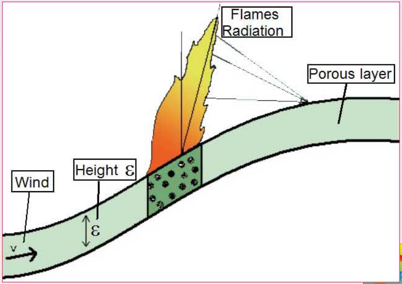
- Time terms
- Convective transport due to the wind
- Diffusion within the vegetation
- Vertical cooling or vertical loss
- Link between T_s and T_g
- Arrhenius type laws for combustion and pyrolysis
- Radiation term $R(T_g)$



Combustion models: A general example. Equations

1. $(1-\phi)\rho_s C_s \frac{\partial T_s}{\partial t} - \nabla(\kappa_s \nabla T_s) = R(T_g) - h(T_s - T_g)$
2. $\phi\rho C(\frac{\partial T_g}{\partial t} + \mathbf{v}\cdot\nabla T_g) - \nabla(\kappa_g\nabla T_g) = -h(T_g-T_s) + qA_g Y_o Y_g e^{-\frac{E_a}{RT_g}} - h_s(T_g - T_\infty)$
3. $\frac{\partial Y_s}{\partial t} = -A_s Y_s e^{-\frac{E_a}{RT_s}}$
4. $\frac{\partial Y_o}{\partial t} + \mathbf{v}\cdot\nabla Y_o - \nabla(\kappa_{oo}\nabla Y_o) = -h_o(Y_o - Y_{o,\infty}) - A_g Y_o Y_g e^{-\frac{E_a}{RT_g}}$
5. $\frac{\partial Y_g}{\partial t} + \mathbf{v}\cdot\nabla Y_g - \nabla(\kappa_{gg}\nabla T_s) = -h_g(Y_g - 0) - A_g Y_o Y_g e^{-\frac{E_a}{RT_g}} + AY_s e^{-\frac{E_a}{RT_s}}$

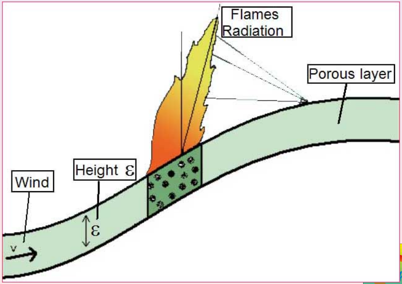
- Time terms
- Convective transport due to the wind
- Diffusion within the vegetation
- Vertical cooling or vertical loss
- Link between T_s and T_g
- Arrhenius type laws for combustion and pyrolysis
- Radiation term $R(T_g)$



Combustion models: A general example. Equations

1. $(1-\phi)\rho_s C_s \frac{\partial T_s}{\partial t} - \nabla(\kappa_s \nabla T_s) = R(T_g) - h(T_s - T_g)$
2. $\phi\rho C(\frac{\partial T_g}{\partial t} + \mathbf{v}\cdot\nabla T_g) - \nabla(\kappa_g \nabla T_g) = -h(T_g - T_s) + qA_g Y_o Y_g e^{-\frac{E_a}{RT_g}} - h_s(T_g - T_\infty)$
3. $\frac{\partial Y_s}{\partial t} = -A_s Y_s e^{-\frac{E_a}{RT_s}}$
4. $\frac{\partial Y_o}{\partial t} + \mathbf{v}\cdot\nabla Y_o - \nabla(\kappa_{oo}\nabla Y_o) = -h_o(Y_o - Y_{o,\infty}) - A_g Y_o Y_g e^{-\frac{E_a}{RT_g}}$
5. $\frac{\partial Y_g}{\partial t} + \mathbf{v}\cdot\nabla Y_g - \nabla(\kappa_{gg}\nabla T_s) = -h_g(Y_g - 0) - A_g Y_o Y_g e^{-\frac{E_a}{RT_g}} + AY_s e^{-\frac{E_a}{RT_s}}$

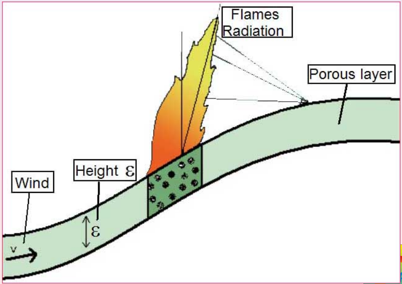
- Time terms
- Convective transport due to the wind
- Diffusion within the vegetation
- Vertical cooling or vertical loss
- Link between T_s and T_g
- Arrhenius type laws for combustion and pyrolysis
- Radiation term $R(T_g)$



Combustion models: A general example. Equations

1. $(1-\phi)\rho_s C_s \frac{\partial T_s}{\partial t} - \nabla(\kappa_s \nabla T_s) = R(T_g) - h(T_s - T_g)$
2. $\phi\rho C(\frac{\partial T_g}{\partial t} + \mathbf{v}\cdot\nabla T_g) - \nabla(\kappa_g \nabla T_g) = -h(T_g - T_s) + qA_g Y_o Y_g e^{-\frac{E_a}{RT_g}} - h_s(T_g - T_\infty)$
3. $\frac{\partial Y_s}{\partial t} = -A_s Y_s e^{-\frac{E_a}{RT_s}}$
4. $\frac{\partial Y_o}{\partial t} + \mathbf{v}\cdot\nabla Y_o - \nabla(\kappa_{oo}\nabla Y_o) = -h_o(Y_o - Y_{o,\infty}) - A_g Y_o Y_g e^{-\frac{E_a}{RT_g}}$
5. $\frac{\partial Y_g}{\partial t} + \mathbf{v}\cdot\nabla Y_g - \nabla(\kappa_{gg}\nabla T_s) = -h_g(Y_g - 0) - A_g Y_o Y_g e^{-\frac{E_a}{RT_g}} + AY_s e^{-\frac{E_a}{RT_s}}$

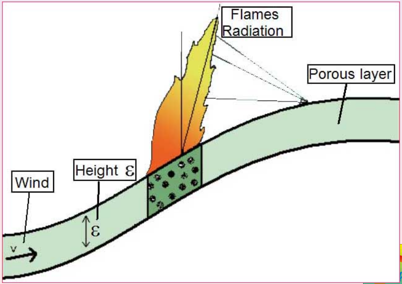
- Time terms
- Convective transport due to the wind
- Diffusion within the vegetation
- Vertical cooling or vertical loss
- Link between T_s and T_g
- Arrhenius type laws for combustion and pyrolysis
- Radiation term $R(T_g)$



Combustion models: A general example. Equations

1. $(1-\phi)\rho_s C_s \frac{\partial T_s}{\partial t} - \nabla(\kappa_s \nabla T_s) = R(T_g) - h(T_s - T_g)$
2. $\phi \rho C(\frac{\partial T_g}{\partial t} + \mathbf{v} \cdot \nabla T_g) - \nabla(\kappa_g \nabla T_g) = -h(T_g - T_s) + qA_g Y_o Y_g e^{-\frac{E_a}{RT_g}} - h_s(T_g - T_\infty)$
3. $\frac{\partial Y_s}{\partial t} = -A_s Y_s e^{-\frac{E_a}{RT_s}}$
4. $\frac{\partial Y_o}{\partial t} + \mathbf{v} \cdot \nabla Y_o - \nabla(\kappa_{oo} \nabla Y_o) = -h_o(Y_o - Y_{o,\infty}) - A_g Y_o Y_g e^{-\frac{E_a}{RT_g}}$
5. $\frac{\partial Y_g}{\partial t} + \mathbf{v} \cdot \nabla Y_g - \nabla(\kappa_{gg} \nabla T_s) = -h_g(Y_g - 0) - A_g Y_o Y_g e^{-\frac{E_a}{RT_g}} + AY_s e^{-\frac{E_a}{RT_s}}$

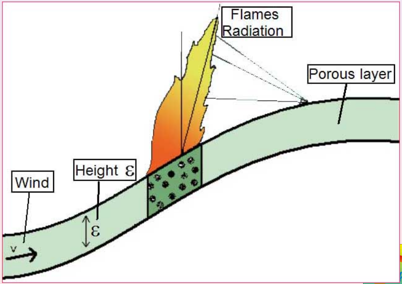
- Time terms
- Convective transport due to the wind
- Diffusion within the vegetation
- Vertical cooling or vertical loss
- Link between T_s and T_g
- Arrhenius type laws for combustion and pyrolysis
- Radiation term $R(T_g)$



Combustion models: A general example. Equations

1. $(1-\phi)\rho_s C_s \frac{\partial T_s}{\partial t} - \nabla(\kappa_s \nabla T_s) = R(T_g) - h(T_s - T_g)$
2. $\phi\rho C(\frac{\partial T_g}{\partial t} + \mathbf{v}\cdot\nabla T_g) - \nabla(\kappa_g \nabla T_g) = -h(T_g - T_s) + qA_g Y_o Y_g e^{-\frac{E_a}{RT_g}} - h_s(T_g - T_\infty)$
3. $\frac{\partial Y_s}{\partial t} = -A_s Y_s e^{-\frac{E_a}{RT_s}}$
4. $\frac{\partial Y_o}{\partial t} + \mathbf{v}\cdot\nabla Y_o - \nabla(\kappa_{oo}\nabla Y_o) = -h_o(Y_o - Y_{o,\infty}) - A_g Y_o Y_g e^{-\frac{E_a}{RT_g}}$
5. $\frac{\partial Y_g}{\partial t} + \mathbf{v}\cdot\nabla Y_g - \nabla(\kappa_{gg}\nabla T_s) = -h_g(Y_g - 0) - A_g Y_o Y_g e^{-\frac{E_a}{RT_g}} + AY_s e^{-\frac{E_a}{RT_s}}$

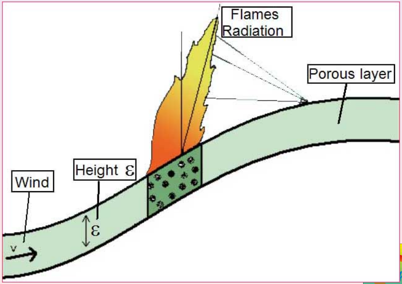
- Time terms
- Convective transport due to the wind
- Diffusion within the vegetation
- Vertical cooling or vertical loss
- Link between T_s and T_g
- Arrhenius type laws for combustion and pyrolysis
- Radiation term $R(T_g)$



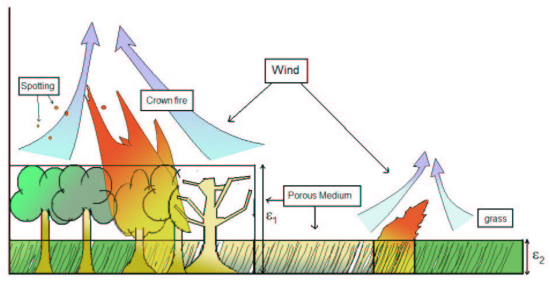
Combustion models: A general example. Equations

1. $(1-\phi)\rho_s C_s \frac{\partial T_s}{\partial t} - \nabla(\kappa_s \nabla T_s) = R(T_g) - h(T_s - T_g)$
2. $\phi\rho C(\frac{\partial T_g}{\partial t} + \mathbf{v}\cdot\nabla T_g) - \nabla(\kappa_g \nabla T_g) = -h(T_g - T_s) + qA_g Y_o Y_g e^{-\frac{E_a}{RT_g}} - h_s(T_g - T_\infty)$
3. $\frac{\partial Y_s}{\partial t} = -A_s Y_s e^{-\frac{E_a}{RT_s}}$
4. $\frac{\partial Y_o}{\partial t} + \mathbf{v}\cdot\nabla Y_o - \nabla(\kappa_{oo}\nabla Y_o) = -h_o(Y_o - Y_{o,\infty}) - A_g Y_o Y_g e^{-\frac{E_a}{RT_g}}$
5. $\frac{\partial Y_g}{\partial t} + \mathbf{v}\cdot\nabla Y_g - \nabla(\kappa_{gg}\nabla T_s) = -h_g(Y_g - 0) - A_g Y_o Y_g e^{-\frac{E_a}{RT_g}} + AY_s e^{-\frac{E_a}{RT_s}}$

- Time terms
- Convective transport due to the wind
- Diffusion within the vegetation
- Vertical cooling or vertical loss
- Link between T_s and T_g
- Arrhenius type laws for combustion and pyrolysis
- Radiation term $R(T_g)$



Numerical Simulation of forest fires in small computers with computation times being a small fraction of the real time



Requirements

Simple models

- Simplified mathematical models
- Basically two dimensional models
- Some physical phenomena will be parametrized.

Realistic models

- Take into account main mechanisms of propagation
- Take into account three dimensional effects

Requirements

Simple models

- Simplified mathematical models
- Basically two dimensional models
- Some physical phenomena will be parametrized.

Realistic models

- Take into account main mechanisms of propagation
- Take into account three dimensional effects

Requirements

Simple models

- Simplified mathematical models
- Basically two dimensional models
- Some physical phenomena will be parametrized.

Realistic models

- Take into account main mechanisms of propagation
- Take into account three dimensional effects

Requirements

Simple models

- Simplified mathematical models
- Basically two dimensional models
- Some physical phenomena will be parametrized.

Realistic models

- Take into account main mechanisms of propagation
- Take into account three dimensional effects

Requirements

Simple models

- Simplified mathematical models
- Basically two dimensional models
- Some physical phenomena will be parametrized.

Realistic models

- Take into account main mechanisms of propagation
- Take into account three dimensional effects

Convection-Diffusion-Radiation simplified models

Require the solution 1D or 2D of Convection-Diffusion-Radiation equations.

layer conducting radiating material

- $\rho C(\frac{\partial u}{\partial t} + \mathbf{V}\nabla u) - (J * u^4 - u^4) + \alpha u = Af(u,y)$
- $\frac{\partial y}{\partial t} = -f(u,y)$

- u Temperature
- y Mass fraction of fuel
- $-(J * u^4 - u^4)$ Non local diffusion (due to radiation)

Local approximation

Estimation of the term $-(J * u^4 - u^4)$

$$
\begin{aligned}
(J * u^4)(x) - u^4 &= \int J(x-y)u^4(y)dy - u^4(x) \\
&= \int J(x-y)(u^4(y) - u^4(x))dy \\
&= \int J(x-y)((u^4)'(x)(y-x) + \frac{1}{2}(u^4)''(x) + \ldots)dy \\
&= -(\int J(z)zdz)(u^4)'(x) + (\int J(z)z^2dz)(u^4)''(x) +
\end{aligned}
$$

If the kernel $J(.)$ is symmetric then

$$-(J * u^4 - u^4) \approx -(\int J(z)z^2dz)(4u^3u')'(x)$$

If the kernel is not symmetric (due to wind which tilt the flame) a convection term appears

$$-(J * u^4 - u^4) \approx \left(\int J(z) z dz\right)(u^4)'(x) - \left(\int J(z) z^2 dz\right)(4u^3 u')'(x)$$

Diffusion approximation

If it is assumed that the absortion mean penetration distance $\frac{1}{a}$ is small compared with the distances over which significant temperatures changes occur, that is the effect of radiation is local (Rosseland approximation)

layer conducting radiating material

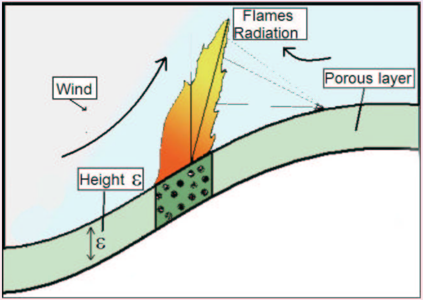
- $\rho C(\frac{\partial u}{\partial t} + \mathbf{V}\nabla u) - \nabla(\frac{16\sigma u^3}{3a_R}\nabla u) + \alpha u = Af(u,y)$
- $\frac{\partial y}{\partial t} = -f(u,y)$

- a_R Rosseland absortion

A Simple Physical Model

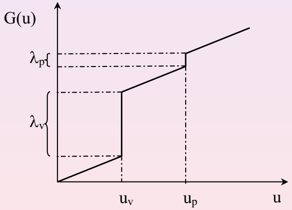
The non dimensional equations governing the fire spread in a region Ω are,

1. $\partial_t e + \alpha u = r(u, y)$
2. $e \in G(u)$
3. $\partial_t y = -g(u)y$



- $r(u, y)$ is the source term
- $G(u)$ is the Enthalpy
- $g(u)y$ is the rate of pyrolysis

$$G(u) = \begin{cases} u & \text{if} \quad u < u_v \\ [u_v \ , \ u_v + \lambda_v] & \text{if} \quad u = u_v \\ u + \lambda_v & \text{if} \quad u_v < u < u_p \\ [u_p + \lambda_v \ , \ u_p + \lambda_v + \lambda_p] & \text{if} \quad u = u_p \\ u + \lambda_v + \lambda_p & \text{if} \quad u > u_p \end{cases}$$



Comments

- Local diffusion has been neglected
- Convection has been neglected but main WIND EFFECTS are considered via radiation model
- Main mechanism of fuel heating is RADIATION (non local diffusion)
- Effects of the WATER CONTENT are considered
- Gas phase is parametrized (The temperature and height of the flame are parameters)

Euler semi-implicit method

Let $\Delta t = t^{n+1} - t^n$ a time step and let y^n, e^n and u^n denote approximations at time step t^n, to the exact solution y, e and u respectively.

We consider a semi-implicit scheme. At each time step we solve,

$$\frac{e^{n+1} - e^n}{\Delta t} + \alpha u^{n+1} = r^n$$

$$e^{n+1} \in G(u^{n+1})$$

$$\frac{y^{n+1} - y^n}{\Delta t} = -y^{n+1} g(u^{n+1})$$

Solving the nonlinear equation in each time step

The multivalued operator $G(.)$ is maximal monotone, then its resolvent $J_\lambda = (Id + \lambda G)^{-1}$ for any $\lambda > 0$ is a well defined univalued operator.

Taking $\lambda = 1/(\alpha \Delta t)$ the algorithm simplifies

1. $u^{n+1} = J_{1/\alpha\Delta t}(\frac{1}{\alpha \Delta t} e^n + \frac{1}{\alpha} r^n)$
2. $e^{n+1} = e^n - \alpha \Delta t u^{n+1} + \Delta t r^n$
3. $y^{n+1} = \frac{y^n}{1 + \Delta t g(u^{n+1})}$

Practical Computation of the Resolvent

$$(\alpha\Delta t\, I + G)\bar{z} \ni \bar{u}$$

- if $(1+\alpha\Delta t\,)u_v < \bar{u} < (1+\alpha\Delta t\,)u_v + \lambda_v$,

  $\bar{z} = u_v$

- if $(1+\alpha\Delta t\,)u_v + \lambda_v < \bar{u} < (1+\alpha\Delta t\,)u_p + \lambda_v$

  $\bar{z} = \frac{\bar{u}-\lambda_v}{1+\alpha\Delta t}$

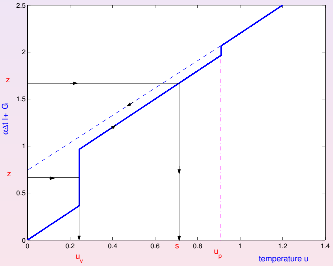
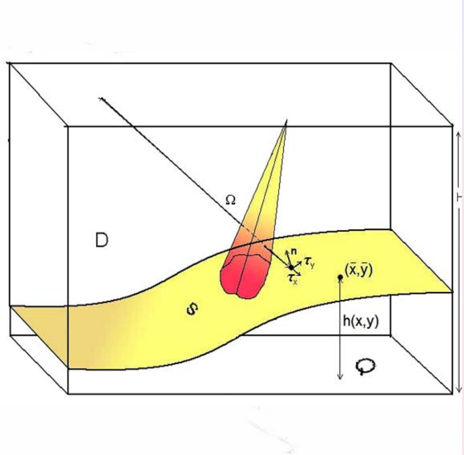


Figure: Computation of $\bar{z}$

Nonlocal radiation

The source term is due to nonlocal radiation: In each point it is caculated summing up the radiation intensity for all directions Ω.



1- Assume as known $i(x,\Omega)$ $\forall\, \Omega$ for a given x

2- By use of polar coordinates at the tangent space of $\bar{\mathbf{x}}$ and defining $\mu = \cos\theta$ and $\gamma = \cos\phi$

$$r(\bar{\mathbf{x}}) = \int i(\bar{\mathbf{x}}, \mathbf{\Omega})\mathbf{\Omega}.\mathbf{n}\; d\omega = \int_{\mu,\gamma} \frac{i_{+}(\bar{\mathbf{x}},\mu,\gamma)\mu}{\sqrt{1-\gamma^2}}\; d\mu d\gamma$$

3- A quadrature may be done with respect to:

$\mu \longrightarrow$ Gauss-Legendre
$\gamma \longrightarrow$ Gauss-Chevichev

Radiation equation

After adimensionalization, the radiation equations in the direction Ω can be written as

$$\boldsymbol{\Omega}.\nabla i + a^* i = \delta(1 + u_g)^4 \quad \text{in } D$$
$$i = 0 \quad \text{on } \partial D \cap \{\mathbf{x};\ \boldsymbol{\Omega}.\mathbf{n} < 0\}$$

The incident energy at a point $\mathbf{x}(x, y, h(x, y))$ of the surface S per unit time and per unit area will be obtained summing up the contribution of all directions $\boldsymbol{\Omega}$

$$r(\mathbf{x}) = \int_{\omega=0}^{2\pi} i(\mathbf{x}, \boldsymbol{\Omega})\boldsymbol{\Omega}.\mathbf{n}\ d\omega \qquad (1)$$

The radiation term r in the energy equation is computed by numerical integration.

Summing up for all the solid angles

$$r(\bar{\mathbf{x}}) = \int_{\theta=0}^{\theta=\pi/2} \int_{\phi=0}^{\phi=2\pi} i(\bar{\mathbf{x}}, \theta, \phi) \cos\theta \sin\theta \; d\theta d\phi =$$

$$\int_{\mu=0}^{\mu=1} \int_{\gamma=-1}^{\gamma=1} \frac{i_+(\bar{\mathbf{x}}, \mu, \gamma)\mu}{\sqrt{1-\gamma^2}} \; d\mu d\gamma \; + \int_{\mu=0}^{\mu=1} \int_{\gamma=-1}^{\gamma=1} \frac{i_-(\bar{\mathbf{x}}, \mu, \gamma)\mu}{\sqrt{1-\gamma^2}} \; d\mu d\gamma \quad (2)$$

$\mu = \cos\theta$, $\gamma = \cos\phi$

i_+ (resp. i_-) stands for the radiation intensity i corresponding to an angle ϕ such that $0 \leq \phi < \pi$ (resp. $\pi \leq \phi < 2\pi$).

The integrals are computed using Gauss-Legendre quadrature with respect μ and Gauss-Chebyshev quadrature with respect γ in order to cope with the singular weight $\frac{1}{\sqrt{1-\gamma^2}}$.

$$r(\bar{\mathbf{x}}) \approx \sum_{k,l} W_{kl}\; i_{+}(\bar{\mathbf{x}}, \mu_k, \gamma_l)\mu_k + \sum_{k,l} W_{kl}\; i_{-}(\bar{\mathbf{x}}, \mu_k, \gamma_l)\mu_k \qquad (3)$$

Numerical solution of the radiation equation

The characteristic line is

$$[0, \xi] \longmapsto \mathcal{R}^3$$

$$\xi \longrightarrow (x(\xi) = \bar{x} + \xi\Omega_1,\ y(\xi) = \bar{y} + \xi\Omega_2,\ z(\xi) = \bar{z} + \xi\Omega_3)$$

On the characteristic, the radiation equation becomes

$$\frac{di}{d\xi} + a^* i = \delta(1 + u_g)^4$$

which can be solved backwards together with the condition

$$\lim_{\xi \to \infty} i(\xi) = 0 \qquad (4)$$

Extension of the temperature field

When integrating the radiation equation we need to know $a^*(\xi(x,y,z))$ and $u_g(\xi(x,y,z)$ so we need to extend the temperature field to the whole domain D.

- If there is NO WIND, we extend the temperature vertically

$$\tilde{u}(x,y,z) = u(x,y,h(x,y))$$

$$h(x,y) < z < h(x,y) + H$$

.

- In the case of WIND CONDITIONS we compute the extended field assuming a convective transport

$$\tilde{u}(x,y,z) = u(x-(z-h(x,y))\frac{V_x}{V_z}, y-(z-h(x,y))\frac{V_y}{V_z}, h(x,y))$$

.

Extension of the temperature field

When integrating the radiation equation we need to know $a^*(\xi(x,y,z))$ and $u_g(\xi(x,y,z)$ so we need to extend the temperature field to the whole domain D.

- If there is NO WIND, we extend the temperature vertically

$$\tilde{u}(x,y,z) = u(x,y,h(x,y))$$

$$h(x,y) < z < h(x,y) + H$$

.

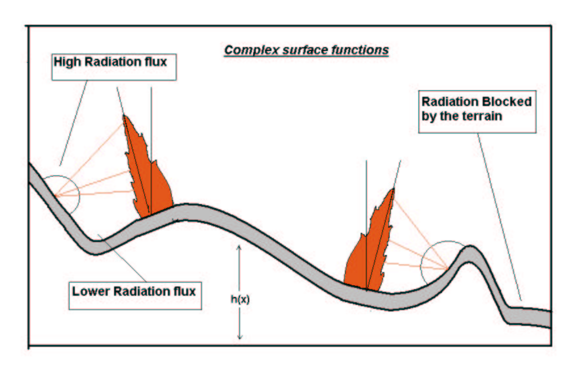
- In the case of WIND CONDITIONS we compute the extended field assuming a convective transport

$$\tilde{u}(x,y,z) = u(x-(z-h(x,y))\frac{v_x}{v_z}, y-(z-h(x,y))\frac{v_y}{v_z}, h(x,y))$$

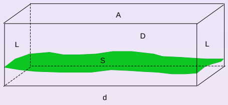
.

The radiation equation and complex geometrie

The method is able to cope with complex geometries



Vertical diffusion model



Gives the horizontal wind field in a 3D domain by the expression

$$V(\mathbf{x}, z) = m(\mathbf{x}, z)\nabla p(\mathbf{x}) + n(\mathbf{x}, z)\nabla \hat{T}(\mathbf{x})$$

1. $m(\mathbf{x}, z) = \frac{1}{2}z^2 - \delta z - \frac{1}{2}h^2(\mathbf{x}) + (\delta + \xi)h(\mathbf{x}) - \xi\delta$
2. $n(\mathbf{x}, z) = -\frac{1}{24}z^4 + \frac{1}{6}\delta z^3 - \frac{1}{3}\delta^3 z + \frac{1}{24}h^4(\mathbf{x}) - \dots$
3. $p(\mathbf{x})$ is a potential function

Vertical diffusion model

The potential $p(\mathbf{x})$ satisfies the following boundary problem

$$-\nabla(a\nabla p) = \nabla(b\nabla \hat{T}) \quad \text{in } d$$
$$a\frac{\partial p}{\partial n} = -b\frac{\partial \hat{T}}{\partial \nu} + (\delta - h)v_m.\nu \quad \text{on} \quad \partial d$$

1. $a(\mathbf{x}) = \frac{1}{3}(\delta - h(\mathbf{x}))^2(3\xi + \delta - h(\mathbf{x}))$
2. $b(\mathbf{x}) = \frac{1}{30}(\delta - h(\mathbf{x})^2\Big(2\delta^2(2\delta + 5\xi) - 2\delta(\delta - 5\xi)h(\mathbf{x}) - (3\delta + 5\xi)h^2(\mathbf{x}) + h^3(\mathbf{x})\Big)$

Optimal control problem

The optimal control problem is caracterised by

- $$\int_\omega a\nabla p(u)\nabla\varphi + \frac{1}{\alpha}\int_{\partial\omega} q\varphi = -\int_\omega b\nabla\hat{T}\nabla\varphi \quad \forall\varphi\in V$$

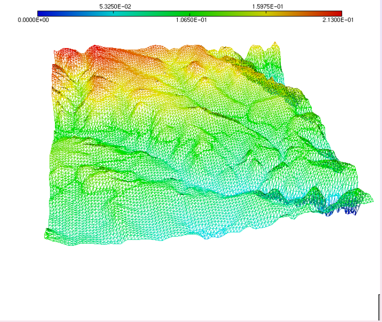
- $$\int_\omega a\nabla q(u)\nabla\psi$$
  $$-\sum_{i=1}^N \int_\omega \rho_\epsilon(x-x_i)(m\nabla p(u)+n\nabla\hat{T}-V_i)m\nabla\psi = 0 \quad \forall\psi\in V$$

- $$u = -\frac{1}{\alpha}q \quad \text{on} \quad \partial\, d$$

An example: Simulation: Cofrentes (Valencia, Spain)

The data

- Zone 5 Km long by 5 Km width
- Average Meteorological wind given.
- The ignition point
- GIS data: Surface orography, vegetation density
- Combustion properties of the fuel: half-life decay.



Data by courtesy of Tecnosylva, S.L. León, Spain

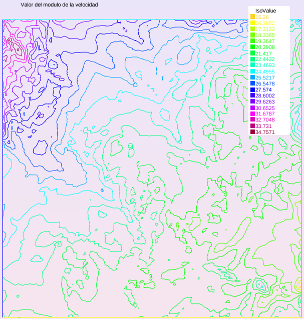


Figure: Wind Module

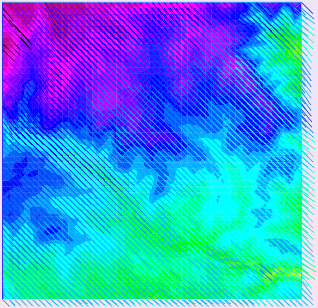


Figure: Wind direction

Evolution of the fire front with Wind

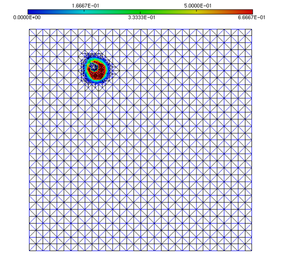


Figure: With wind at time 33 minutes

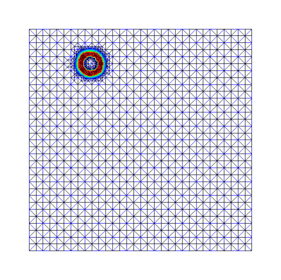


Figure: Without wind at time 33 minutes

Evolution of the fire front with Wind

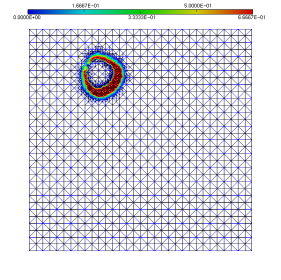


Figure: With wind at time 66 minutes

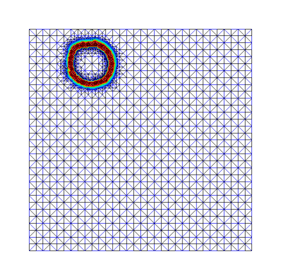


Figure: Without wind at time 66 minutes

Evolution of the fire front with Wind

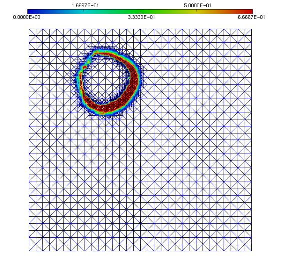


Figure: With wind at time 99 minutes

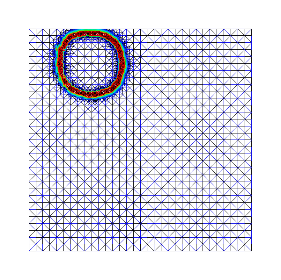


Figure: Without wind at time 99 minutes

Evolution of the fire front with Wind

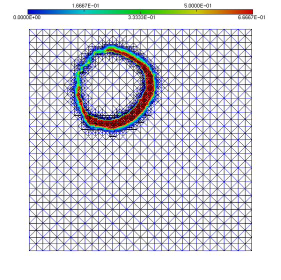


Figure: With wind at time 132 minutes

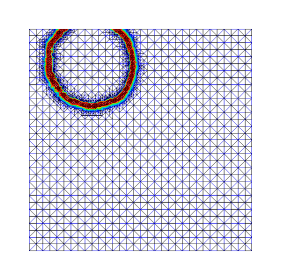


Figure: Without wind at time 132 minutes

Evolution of the fire front with Wind

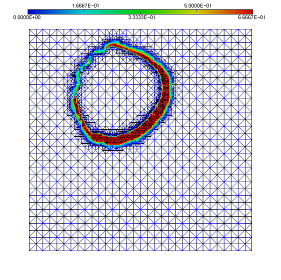


Figure: With wind at time 165 minutes

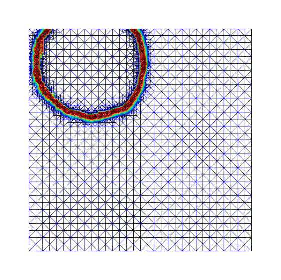


Figure: Without wind at time 165 minutes

Evolution of the fire front with Wind

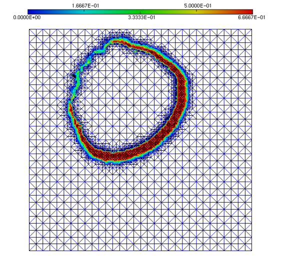


Figure: With wind at time 198 minutes

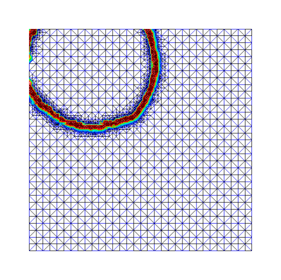


Figure: Without wind at time 198 minutes

Evolution of the fire front with Wind

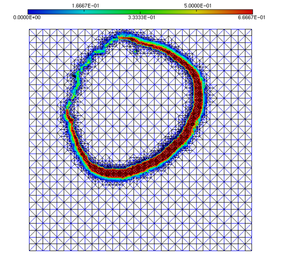


Figure: With wind at time 231 minutes

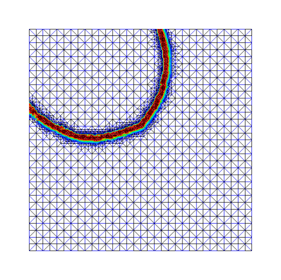


Figure: Without wind at time 231 minutes

Evolution of the fire front with Wind

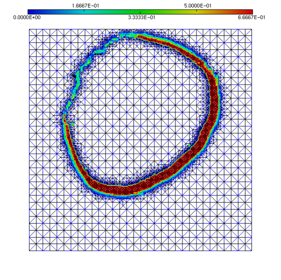


Figure: With wind at time 264 minutes

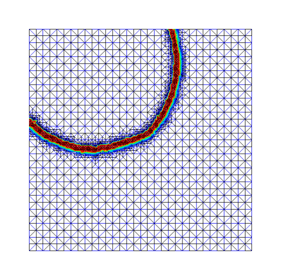


Figure: Without wind at time 264 minutes

Evolution of the fire front with Wind

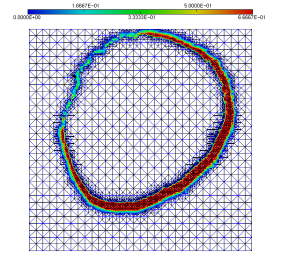


Figure: With wind at time 297 minutes

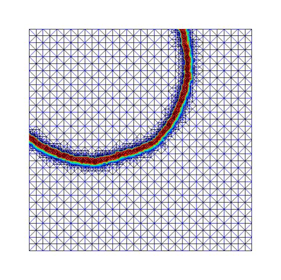


Figure: Without wind at time 297 minutes

3D representation

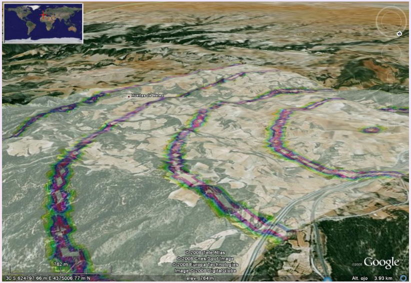


Figure: Without wind

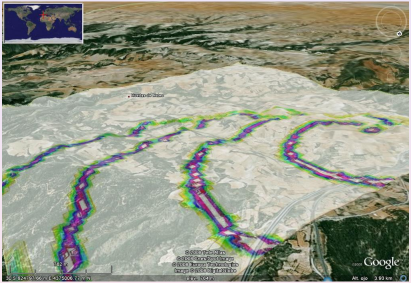
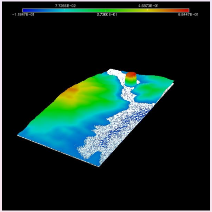


Figure: With wind

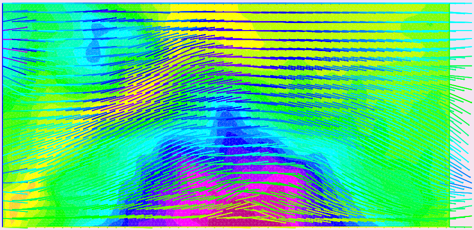
An example: Simulation of a fire in a river basin

The data

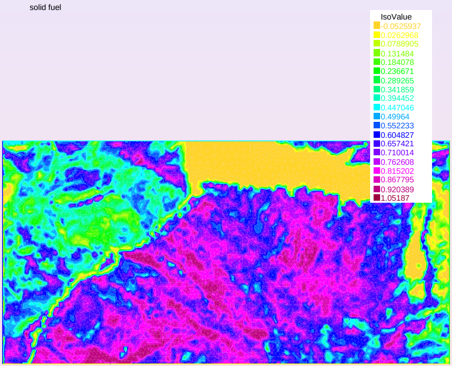
- Zone 6 Km long by 3 Km width
- Meteorological wind given in several points.
- The ignition point
- GIS data: Surface orography, vegetation density
- Combustion properties of the fuel: half-life decay.



Wind: wind field V



Initial Fuel Density



Main physical parameters

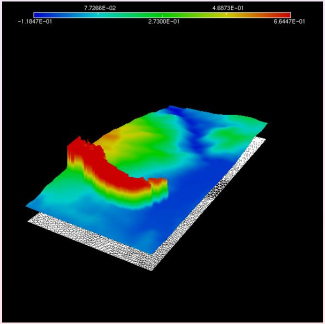
- Zone 6 Km long by 3 Km width
- Water content: 2%
- Half-time decay: 700 seconds
- Flame height: 20 meters
- Flame temperature 1225C
- Wind velocity (1st): NO WIND
- Wind velocity (2nd): $10m/s \div 20m/s$

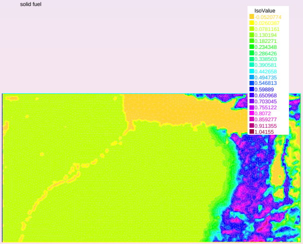
Main numerical data

- Mesh size: maximum size 100 meters, minimum size 50 meters
- number of uknowns per variable: 31774
- Time step: 200 seconds
- Numerical integration points for radiation: $8 \times 2 = 16$

Computation results

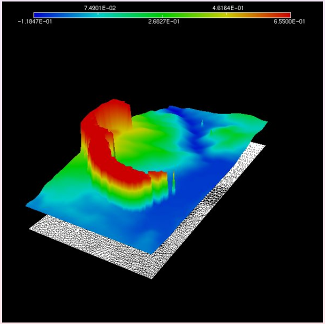
- Real time of the simulation ≈ 5 hours 30 minutes
- Computation time in my laptop: 25 minutes
- Mean fire front velocity: 650 m/h (no wind)

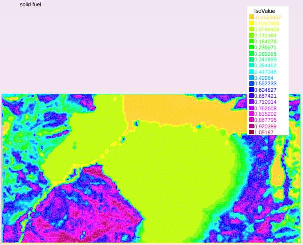




Computation results

- Real time of the simulation ≈ 5 hours 30 minutes
- Computation time in my laptop: 25 minutes
- Mean fire front velocity: 710 m/h (wind)





Evolution of a fire in the Ebro basin

SHOW ANIMATIONS